Optimal Deposit Insurance*

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Abstract

This paper characterizes the optimal level of deposit insurance (DI) when bank runs are possible. In a wide variety of environments, the optimal level of DI only depends on three sufficient statistics: the sensitivity of the likelihood of bank failure with respect to the level of DI, the utility loss caused by bank failure (which is a function of the drop in depositors' consumption) and the direct social costs of intervention in the case of bank failure, which directly depend on the unconditional probability of bank failure, the marginal cost of public funds, and the illiquidity/insolvency status of banks. Because banks do not internalize the fiscal implications of their actions, changes in the behavior of competitive banks induced by varying the level of DI (often referred to as moral hazard) only affect the level of optimal DI directly through a fiscal externality, but not independently. We characterize the wedges that determine the optimal ex-ante regulation (which can be mapped to deposit insurance premia) and discuss the practical implications of our framework in the context of US data.

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1 Introduction

Bank failures have been a recurrent phenomenon in the United States and in many other countries throughout modern history. A sharp change in the United States banking system occurs with the introduction of federal deposit insurance in 1934, which dramatically reduced the number of bank failures. For reference, more than 13,000 banks failed between 1921 and 1933, and 4,000 banks failed only in 1933. In contrast, a total of 4,057 banks have failed in the United States between 1934 and 2014.[1] As of today, deposit insurance remains a crucial pillar of financial regulation and represents the most salient form of explicit government guarantees to the financial sector.

Despite its success reducing bank failures, deposit insurance entails fiscal costs when it has to be paid and affects the ex-ante behavior of market participants (these behavioral responses are often referred to as moral hazard). Hence, in practice, deposit insurance only guarantees a fixed level of deposits. As we show in figure 6 in the appendix, this level of coverage has changed over time. Starting from the original $2,500 per account in 1934, the insured limit in the US is $250,000 dollars since 2008. A natural question to ask is how the level of this guarantee should be determined to maximize social welfare. In particular, what is the optimal level of deposit insurance? Are $250,000, the current value in the US, or €100,000, the current value in most European countries, the optimal levels of deposit insurance for these economies? Which variables ought to be measured to optimally determine the level of deposit insurance coverage in a given economy?

This paper provides an analytical characterization, written as a function of observable or potentially recoverable variables, which directly addresses those questions. Although there has been progress in understanding theoretical tradeoffs related to deposit insurance, a general framework that incorporates the most relevant tradeoffs which can be used to provide explicit quantitative guidance to answer these questions has been missing. With this paper, we provide a first step in that direction.

We initially derive the main results of the paper in a version of the canonical model of bank runs of Diamond and Dybvig (1983). In our basic framework, competitive banks set the interest rate on a deposit contract to share risks between early and late depositors in an environment with aggregate uncertainty about the profitability of banks investments. Due to the demandable nature of the deposit contract, depending on the aggregate state, both fundamental-based and panic-based bank failures are possible. Mimicking actual deposit insurance arrangements, we assume that deposits are guaranteed by the government up to a deposit insurance limit of

[1] All these figures come from the FDIC Historical Statistics on Banking and FDIC (1998). Weighting bank failures by the level of banks assets or correcting by the total number of banks still generates a significant discontinuity on the level of bank failures after the introduction of deposit insurance. See Demirgüç-Kunt, Kane and Laeven (2014) for a recent description of deposit insurance systems around the world.

δ dollars and then focus on the implications for social welfare of varying δ.[2] Our positive analysis shows that increasing δ, holding the deposit rate offered by banks constant, reduces the likelihood of bank failure, which is an important input for our normative analysis. We assume throughout that any transfer of resources associated with deposit insurance payments entails a fiscal cost, given by the marginal value of public funds.

After studying how varying δ affects equilibrium outcomes, we focus on the welfare implications of such policy. The best way to present our results is by describing the determinants of the optimal level of deposit insurance δ^*. The optimal deposit insurance level can be written as a function of a few sufficient statistics in the following way:

$$\delta^* = \frac{\text{Sensitivity of bank failure probability to a proportional change in } \delta \times \text{Drop in depositors consumption}}{\text{Probability of bank failure} \times \text{Expected marginal social cost of intervention in case of bank failure}} \quad (1)$$

Equation (1) embeds the key tradeoffs regarding the optimal determination of deposit insurance. On the one hand, when a marginal change in δ greatly reduces the likelihood of bank failure, at the same time that the drop in depositors consumption caused by a bank failure is large, it is optimal to set a high level of deposit insurance. On the other hand, when bank failures are frequent and when the social cost of ex-post intervention associated with them — for instance, because it is very costly to raise resources through distortionary taxation — is large, it is optimal to set a low level of deposit insurance. In addition to characterizing the optimal level of deposit insurance, we also provide a directional test that determines whether it is optimal to increase or decrease the level of coverage starting from its current level. Given our sufficient statistic approach, this is our most general and robust result.

The upshot of our precise formulation — introduced in propositions 1 and 2 in the text — is that we can directly observe or recover the different variables that determine δ^*. This implies that our formula can be calibrated and directly used in practice, providing direct guidance to policymakers on which variables ought to be measured to determine the optimal level of deposit insurance. Once the variables in equation (1) are known, the policymaker does not need any other information to set the optimal level of deposit insurance.

Our characterization allows us to derive a number of theoretical results. First, we show that banks behavioral responses to the policy (often referred to as moral hazard) only affect directly our optimal policy formulas through a fiscal externality that increases the social marginal cost of the intervention in case of bank failure.[3] This result seems to go against current mainstream

[2]We do not explicitly model the possibility that a given depositor may have multiple accounts in different banks by taking the initial level of deposits in a given bank as given. However, as long as there exists a cost of switching/opening deposit accounts to make deposit choices not infinitely inelastic — which is consistent with the evidence by Egan, Hortaçsu and Matvos (2014) — our main results remain valid. See Shy, Stenbacka and Yankov (2015) for a model in which depositors can open multiple deposit accounts.

[3]As in the public finance literature, see e.g. Hendren (2013), we use term fiscal externality to refer to the social

thinking, which emphasizes the role of moral hazard as the main welfare loss created by having a deposit insurance system. Our results do not contradict that view. We simply argue that the behavioral responses induced by varying the level of deposit insurance are subsumed into the sufficient statistics we identify. In other words, high insurance levels can induce banks to make decisions that will increase the likelihood and severity of bank failures, but only its effects through the fiscal externality that we identify have a first-order effect on welfare. Our analysis contributes to understand which precise components of banks behavioral responses have a first-order effect on social welfare.

Second, we show how in an environment in which banks never fail and government intervention is never required in equilibrium, it is optimal to fully guarantee deposits. This result, which revisits the classic finding by Diamond and Dybvig (1983), follows directly from equation (1) when the probability of bank failure — in the denominator — tends towards zero.

Third, we show that providing deposit insurance in states in which banks are illiquid but profitable reduces the expected marginal social cost of intervention. Intuitively, higher deposit insurance coverage increases banks' deposit levels and reduces the positive net present value investments that must be liquidated, which generates a first-order welfare gain in those states.

Finally, we show that social welfare is decreasing in the level of deposit insurance for low levels of coverage. Intuitively, low levels of deposit insurance do not have the benefit of eliminating bank runs, but still generate the fiscal cost of having to pay for deposit insurance in case of bank failure.

Our framework also allows us to explore the optimal determination of ex-ante regulation, which in practice corresponds to optimally setting deposit insurance premia or deposit rate regulations. In particular, we show that the optimal ex-ante regulation forces banks to internalize the fiscal externalities induced by their behavioral responses. We show that, in general, the optimal ex-ante regulation involves restricting the behavior of banks regarding both their asset and liability choices. We further contribute by characterizing the wedges that banks must face when the optimal ex-ante regulation is implemented also in the form of sufficient statistics. We also make a sharp distinction between the corrective and revenue raising roles of ex-ante regulations.

The results of our basic model extend to more general environments. First, we allow depositors to have a consumption-savings decision and portfolio decisions. We show that our optimal policy formulas remain unchanged: any behavioral responses to policy along these dimensions are captured by the identified sufficient statistics. Second, we allow banks to have an arbitrary number of investment opportunities, with different liquidity and return properties. This possibility modifies the social cost of intervention in case of bank failure, introducing a new fiscal externality term. Third, we show that our sufficient statistics remain invariant

resource cost caused by the need to distortionarily raise public funds.

to introducing alternative equilibrium selection mechanisms, like global games. Fourth, we introduce general equilibrium effects and show that the optimal deposit insurance level features a macro-prudential correction when ex-ante regulation is not perfect. Finally, we show that our results extend to the case in which depositors have different levels of deposit holdings.

To show the applicability of our results in practice, we conduct an exercise using our optimal deposit insurance formula with US data. We rationalize the 2008 policy change while recovering the implied bank failure sensitivities to changes in the level of deposit insurance. Our quantitative results illustrate how to apply our framework, but only further work on measurement can provide direct guidance to policymakers.

Related Literature

This paper is directly related to the literature on banking and bank runs that follows Diamond and Dybvig (1983), as Cooper and Ross (1998), Rochet and Vives (2004), Goldstein and Pauzner (2005), Allen and Gale (2007), Uhlig (2010) or Keister (2012). As originally pointed out by Diamond and Dybvig (1983), bank runs can be prevented by modifying the trading structure, in particular by suspending convertibility, or by introducing deposit insurance. A sizable literature on mechanism design, like Peck and Shell (2003), Green and Lin (2003) or Ennis and Keister (2009), among others, has focused on the optimal design of contracts to prevent runs. Taking the contracts used as given, we focus instead on the optimal determination of the deposit insurance limit, which happens to be a policy measure that has been implemented in most modern economies.

The papers by Merton (1977), Kareken and Wallace (1978), Calomiris (1990), Chan, Greenbaum and Thakor (1992), Matutes and Vives (1996), Freixas and Rochet (1998), Freixas and Gabillon (1999), Cooper and Ross (2002), Duffie et al. (2003), Allen, Carletti and Leonello (2011) and Allen et al. (2014) have explored different dimensions of the deposit insurance institution, in particular the possibility of moral hazard and the determination of appropriately priced deposit insurance for an imperfectly informed policymaker. However, most of the literature that studies deposit insurance arrangements has been essentially theoretical and unable to provide practical guidelines to policymakers. In this paper, we provide a unified framework which embeds the main tradeoffs that determine the optimal deposit insurance policy, while providing practical guidance on which variables ought to be measured to set the level of deposit insurance optimally. Our approach crucially relies on characterizing optimal policies as a function of observables.

Our emphasis on measurement is related to a growing quantitative literature on the implications of bank runs and deposit insurance. Demirgüç-Kunt and Detragiache (2002), Ioannidou and Penas (2010), Iyer and Puri (2012) have studied from a reduced form empirical approach the effects of deposit insurance policies. Egan, Hortaçsu and Matvos (2014) have

explored quantitatively different regulations in the context of a rich empirical structural model of deposit choice. Using a macroeconomic perspective, Gertler and Kiyotaki (2013) and Kashyap, Tsomocos and Vardoulakis (2014) have quantitatively assessed, by simulation, the convenience of guaranteeing banks deposits, but they have not characterized optimal policies.

Methodologically, our normative approach and its sufficient statistic implementation relates to the classic normative work in public finance. Diamond (1998), Saez (2001), Chetty (2009) or Hendren (2013) are a few relevant examples. Matvos (2013) follows a similar approach to measure the benefits of contractual completeness. In a macro-finance context, Davila (2015) uses a related approach to optimally determine the level of bankruptcy exemptions. More broadly, our contribution to the theory of measurement belongs to the nascent literature that seeks to inform financial regulation by designing adequate measurement systems for financial markets, recently synthesized in Brunnermeier and Krishnamurthy (2014) and Haubrich and Lo (2013).

The remainder of this paper is organized as follows. Section 2 lays out the basic framework and characterizes the behavior of the economy for a given level of deposit insurance. Section 3 presents the normative analysis, first characterizing the optimal level of deposit insurance when the policymaker cannot regulate the banks' ex-ante decisions and then when it has instruments to regulate them. Section 4 extends the results in several dimensions and section 5 illustrate how to relate the theoretical results to US data. Section 6 concludes. All proofs, detailed derivations and illustrations, as well as a numerical example, are in the appendix.

2 Basic framework

This paper develops a framework to determine the optimal level of deposit insurance coverage. First, we present our main results in a stylized model of bank runs. Section 4 shows that our insights extend naturally to richer environments.

2.1 Environment

Our model builds on Diamond and Dybvig (1983). Time is discrete, there are three dates $t = 0, 1, 2$ and a single type of consumption good (dollar), which serves as numeraire. There is a continuum of aggregate states at date 1, denoted by $s \in [\underline{s}, \bar{s}]$, which become common knowledge to all agents in the economy, but not to the policymaker.

Depositors' preferences There is a unit measure of ex-ante identical depositors, indexed by i. At date 1, depositors privately learn whether they are of the early or the late type. Early types only derive utility from consuming at date 1, while late types only derive utility from consuming at date 2. The fraction of early types is λ and the fraction of late types is $1 - \lambda$ (we

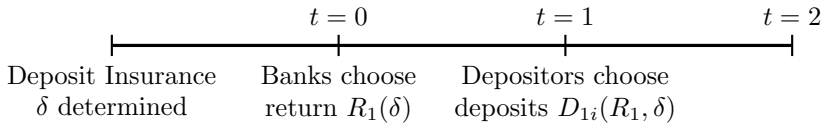

Figure 1: Timeline of choices

assume that a law of large number holds).[4]

Hence, depositors ex-ante utility is given by

$$\mathbb{E}_s \left[\lambda U \left(C_{1i} \left(s \right) \right) + \left(1 - \lambda \right) U \left(C_{2i} \left(s \right) \right) \right],$$

where $C_{1i}(s)$ and $C_{2i}(s)$ respectively denote the expected consumption of depositor i at dates 1 and 2 for a given realization of the aggregate state s.[5] Depositors' flow utility $U(\cdot)$ satisfies standard regularity conditions: $U'(\cdot) > 0$, $U''(\cdot) < 0$ and $\lim_{C \to 0} U'(C) = \infty$.[6] Figure 1 illustrates the timeline of choices.

Depositors' endowments/technology Depositors have an initial endowment $D_0 > 0$ of the consumption good, which they deposit in banks. At date 1, early depositors receive an exogenous endowment $Y_1 > 0$, which, for simplicity, does not depend on the aggregate state. At date 2, late depositors receive an exogenous stochastic endowment $Y_2(s) > 0$ and pay taxes $T_2(s)$ as described below. These exogenous endowments at dates 1 and 2 capture the payoffs on the rest of the portfolios held by depositors — we explicitly model alternative investment opportunities for depositors in section 4.

At date 1, after learning their type, depositors can change their balance of demand deposits by choosing $D_{1i}(s)$: this is the only choice variable for depositors. We also assume that there is an iid sunspot at date 1 for every realization s of the aggregate state — this becomes relevant later on when dealing with multiple equilibria.

We assume that depositors have access to a storage technology between dates 1 and 2 that earns a gross return of $\eta \leq 1$. For all purposes, we always take the limit $\eta \to 1$. As it will become clear, this assumption, which captures the convenience of using bank deposits, makes optimal for late depositors to leave in the bank at date 1 an amount of deposits greater or equal to the level of deposit insurance.

[4] In previous versions of this paper, we allowed for the fraction of early depositors $\lambda(s)$ to vary with the aggregate state, introducing a second source of aggregate risk, without affecting our results.

[5] By assuming that depositors only care about their expected consumption at date 2, we purposefully focus on aggregate efficiency, without having to take care of distributional/risk-sharing concerns among ex-post heterogeneous depositors. It can be shown that our optimal policy results are identical to those in the more general case up to a first-order.

[6] Unlike many bank run models, our model remains well-behaved even when depositors' utility satisfies an Inada condition, because depositors have external sources of income.

Banks' investment technology At date 0, banks have access to a productive technology with the following properties. Every unit invested at date 0, if liquidated at date 1, yields one unit of consumption good for any realization of s. Every unit of investment not liquidated at date 1, yields $\rho_2(s) \geq 0$ units of consumption good at date 2, depending on the realization of s. For simplicity, we assume that banks do not have access to a storage technology at date 1.[7]

We further assume that $\rho_2(s)$ is continuous and increasing in s. High realizations of s correspond to states in which banks are more profitable and vice versa.

Deposit contract The only contract available to depositors is a deposit contract, which takes the following form. A depositor who deposits his endowment at date 0 is promised a noncontingent gross return $R_1 = 1 + r_1$, which can be withdrawn on demand at date 1. Hence, a depositor that deposits D_0 at date 0 is entitled to withdraw up to $R_1 D_0$ deposits at date 1. For simplicity, no interest accrues between dates 1 and 2.

The rate R_1 is set at date 0 by a unit measure of perfectly competitive banks, which make zero profits in equilibrium due to free entry. The actual payoff received by a given depositor at either date 1 or date 2 depends on the returns to bank investments, the behavior of all depositors, and the level of deposit insurance — as described below. As it is assumed in models that follow Diamond and Dybvig (1983), depositors receive all remaining proceeds of bank investments at date 2.

It is useful to introduce the variable $\Delta_i(s)$, which represents the amount of deposits withdrawn by depositor i, formally defined as

$$\Delta_i(s) \equiv R_1 D_0 - D_{1i}(s)$$

We assume that banks must follow a sequential service constraint: banks pay the amount withdrawn $\Delta_i(s)$ to every depositor until they run out of funds. For simplicity, we also assume that early depositors are always repaid first.[8]

Depositors can withdraw funds at date 1 or leave them in the bank, but they cannot add new funds. This restricts depositors' choices to $D_{1i}(s) \in [0, R_1 D_0]$ or, equivalently, $\Delta_i(s) \in [0, R_1 D_0]$. When $\Delta_i(s) > 0$, depositors withdraw a strictly positive fraction of deposits at date 1. When $\Delta_i(s) = 0$, depositors leave their deposit balance untouched. Aggregate net withdrawals are

[7] Any storage technology available to banks at date 1 puts a lower bound on the effective return of investments between date 1 and date 2. Alternatively, we can assume that the return on the storage technology available to banks equals $\rho_2(s)$.

[8] This restriction requires that banks always have enough funds to repay early depositors, that is, $R_1 < \frac{1}{\lambda}$. We must verify that this condition is satisfied in equilibrium for any parameter configuration. Extending the results to the case in which this condition does not hold is tedious but straightforward. Ensuring that early depositors always get paid further allows us to focus on aggregate efficiency without having to take care of distributional concerns among ex-post heterogeneous depositors. In this regard, we depart from the original model of Diamond and Dybvig (1983).

therefore given by

$$\Delta(s) = \int \Delta_i(s)\, di = R_1 D_0 - \int D_{1i}(s)\, di$$

Deposit insurance The level of deposit insurance δ, measured in dollars (units of the consumption good), is the single instrument available to the planner. It is modeled to mimic actual deposit insurance policies: in any event, depositors are guaranteed the promised return on their deposits up to an amount δ, for any realization of the aggregate state s. The level of deposit insurance, which can take any value $\delta \geq 0$ and can be paid either at date 1 or at date 2, is chosen under commitment at date 0 through a planning problem.

For now, no other ex-ante regulation is allowed. Therefore, any funds disbursed to pay for deposit insurance must be raised through taxes at date 2. We denote the fiscal shortfall generated by the deposit insurance system at date 2 in state s by $T_2(s)$. We assume that, for any dollar that needs to be raised through taxes, there is a resource loss of $\kappa \geq 0$ dollars, which represents the marginal cost of public funds.[9] We also assume that, whenever deposit insurance has to actually be paid, the deposit insurance authority is only able to recover a fraction $\chi \in [0,1)$ of any resources held by the banks. This captures the costs of managing and liquidating banks by the deposit insurance authority.

Budget constraints Therefore, given our assumptions, the consumption of a given early depositor at date 1 must satisfy

$$C_{1i}(s) = \Delta_i(s) + Y_1, \qquad i = \text{early},$$

where $\Delta_i(s)$ is a choice variable for early depositors.

Analogously, taking as given the actions of other depositors, the expected consumption of a given late depositor at date 2 in state s can be expressed as

$$C_{2i}(s) = \begin{cases} \alpha_1(s)\left(\eta\left(R_1 D_0 - D_{1i}(s)\right) + \min\left\{D_{1i}(s), \delta\right\}\right) + \left(1-\alpha_1(s)\right)\min\left\{R_1 D_0, \delta\right\} + Y_2(s) - T_2(s), & \text{Failure at date 1} \\ \eta\left(R_1 D_0 - D_{1i}(s)\right) + \min\left\{D_{1i}(s), \delta\right\} + \alpha_{2R}(s)\max\left\{D_{1i}(s) - \delta, 0\right\} + Y_2(s) - T_2(s), & \text{Failure at date 2} \\ \eta\left(R_1 D_0 - D_{1i}(s)\right) + \alpha_{2N}(s) D_{1i}(s) + Y_2(s) - T_2(s), & \text{No Bank Failure} \end{cases}$$

where the scalars $\alpha_1(s) \in [0,1)$, $\alpha_{2R}(s) \in [0,1)$, and $\alpha_{2N}(s) \geq 1$ represent equilibrium objects that depend on the actions of other depositors through the total level of available funds $\rho_2(s)\left(D_0 - \Delta(s)\right)$ at date 2, defining three regions, as follows

$$\text{if} \begin{cases} \rho_2(s)\left(D_0 - \Delta(s)\right) < 0, & \text{Failure at date 1} \\ 0 \leq \rho_2(s)\left(D_0 - \Delta(s)\right) < \int D_{1i}(s)\, di, & \text{Failure at date 2} \\ \rho_2(s)\left(D_0 - \Delta(s)\right) \geq \int D_{1i}(s)\, di, & \text{No Bank Failure} \end{cases} \tag{2}$$

[9] The net marginal cost of public funds, κ, measures the loss incurred in raising additional government revenues — see Dahlby (2008) for an extensive discussion.

From the perspective of an individual depositor, there are three different scenarios. First, when $D_0 - \Delta(s) < 0$, banks are not able to satisfy their withdrawals at date 1, what forces them to liquidate all their investments. Hence, depositors who decide to withdraw early, find that they can only effectively do so with probability $\alpha_1(s) < 1$, determined in equilibrium. With probability $1 - \alpha_1(s)$, depositors do not manage to withdraw any funds at date 1 and only have access to the proceeds from deposit insurance, which correspond to the minimum between their initial deposits $R_1 D_0$ (since they where unable to withdraw any funds) and the level of deposit insurance δ. In this situation, banks fails at date 1.

Second, when the level of date 1 withdrawals does not force banks to fully liquidate their investments, but is such that banks do not have enough resources to pay back their obligations at date 2, late depositors consumption is analogous to the previous case, but as if $\alpha_1(s) = 1$ and correcting for the amount of deposits withdrawn at date 1. Depositors can withdraw as much as they wish at date 1, but they only have access to the minimum between their remaining balance D_{1i} and the level of deposit insurance δ, although they may receive additional resources at a rate α_{2R} if they have claims above δ and banks have sufficient funds. In this situation, banks fail at date 2.

Third, when banks have enough resources to pay depositors more than the level of deposit insurance at date 2, depositors receive a positive net return on their deposits. In this situation, there is no bank failure and no intervention is required.

Equilibrium A symmetric equilibrium, for a given level of deposit insurance δ, is defined as consumption allocations $C_{1i}(s)$ and $C_{2i}(s)$, deposit choices $D_{1i}(s)$, and a return on deposits R_1, such that depositors maximize their utility, given that other depositors behave optimally and banks competitively set R_1 by maximizing depositors utility while making zero profit. We restrict our attention to symmetric equilibria.

Remarks about the environment Before characterizing the equilibrium, we would like to emphasize the two key features of our environment.

First, following most of the literature on bank runs, we take the noncontingent nature of deposits and its demandability as primitives. With this, we depart from the approach that sees deposit contracts as the choice of a mechanism. The upside of our approach is that we can map banks' choices to observable variables, like deposit rates, as opposed to focusing on more abstract assignment procedures.

Second, with respect to the policy instrument, we restrict our attention to a single policy instrument: the amount of deposit insurance coverage. Therefore, we are solving a second-best problem, in the Ramsey tradition. More general policy responses, either explicit or implicit and potentially state contingent, for instance, lender-of-last-resort policies, can bring social welfare closer to the first-best. Even when those policies are available, independently of whether they

are chosen optimally, our main characterization in this paper and all the insights associated with it remain valid as long as they are not able to restore the first-best. We work under the assumption of full commitment throughout.

The two main departures from Diamond and Dybvig (1983) are the presence of aggregate risk regarding bank profitability, which is crucial for our results, and the slightly different timing assumptions regarding the consumption patterns of early and late types, which allow us to simplify the model.

2.2 Equilibrium characterization

For a given level of deposit insurance δ, we characterize the equilibrium of the economy backwards. We first characterize the optimal choice by depositors at date 1 and then study the date 0 choices made by banks. Finally, we solve the planning problem that determines δ^*.

Early depositors Given our assumptions, it is optimal for early depositors to withdraw all their deposits at date 1 and set $D_{1i}(s) = 0$, $\forall s$. Hence early depositors always consume in equilibrium

$$C_{1i}(s) = R_1 D_0 + Y_1, \quad \forall s,\ i = \text{early}$$

Late depositors Late depositors, who only consume at date 2 and have an imperfect storage technology, would in principle prefer to keep their deposits within the banks until date 2. However, they may not receive the total promised amount $R_1 D_0$ if the bank doesn't have enough funds.

We show that only two deposit choices can be optimal for late depositors: a) leave enough deposits so as to receive the full amount of deposit insurance, or b) keep all their deposits in the bank. Formally, we show that $C_{2i}(s)$ is either increasing or decreasing in $D_{1i}(s)$ in all three possible scenarios. Formally, $\frac{dC_{2i}(s)}{dD_{1i}(s)}$ is given by

$$\frac{dC_{2i}(s)}{dD_{1i}(s)} = \begin{cases} \alpha_1(s)\left(\mathbb{I}\left[D_{1i} \leq \delta\right] - \eta\right), & \text{Failure at date 1} \\ \mathbb{I}\left[D_{1i} \leq \delta\right] + \alpha_{2R}\mathbb{I}\left[D_{1i} > \delta\right] - \eta, & \text{Failure at date 2} \\ \alpha_{2N}(s) - \eta, & \text{No Bank Failure} \end{cases}$$

where we use $\mathbb{I}[\cdot]$ to denote an indicator function.

When banks fail at date 1 and $D_{1i}(s) \leq \delta$, $\frac{dC_{2i}(s)}{dD_{1i}(s)} = \alpha_1(s)(1-\eta) > 0$, which is strictly positive. In that case, it is optimal for an individual depositor to increase the level of deposits in the bank at date 1. When banks fail at date 1 but $D_{1i}(s) > \delta$, $\frac{dC_{2i}(s)}{dD_{1i}(s)} = -\alpha_1(s)\eta < 0$, which is strictly negative. In that case, it is optimal for an individual depositor to decrease the level of deposits in the bank at date 1. Both observations imply that it is optimal to precisely choose δ as the level of deposits. The exact same logic applies to the case in which banks fail at date 2.

When there is no bank failure, $\frac{dC_{2i}(s)}{dD_{1i}(s)} = 1 - \eta > 0$, which is strictly positive. In that case, it is optimal for an individual depositor to keep all deposits in the bank at date 1.

Given this result, denoting by $D_1(s)$ the deposit choice of late depositors, there are two candidates for symmetric equilibria

$$D_1(s) = \begin{cases} \min\{\delta, R_1 D_0\}, & \text{Failure equilibrium} \\ R_1 D_0, & \text{No Failure equilibrium} \end{cases}$$

To formally establish that these two deposit choices are equilibria, we must guarantee that the optimal behavior of depositors is consistent with the determination of the three possible scenarios in equation (2). In a symmetric equilibrium in which all late depositors choose $D_1(s)$, the level of resources available per individual late depositor at date 2 is given by

$$\frac{\rho_2(s)(D_0 - \Delta(s))}{1-\lambda} = \rho_2(s)\left(D_1(s) - \frac{r_1 D_0}{1-\lambda}\right) \tag{3}$$

Figure 7 in the appendix illustrates this relation, for given values of $D_1(s)$. Using this expression, we can determine the threshold level of deposits such that banks fail at date 1, given by

$$\frac{r_1 D_0}{1-\lambda}$$

If $D_1(s) < \frac{r_1 D_0}{1-\lambda}$, banks do not have enough funds at date 1 to meet depositors demands. When $D_1(s) \geq \frac{r_1 D_0}{1-\lambda}$, banks have enough funds at date 1 to satisfy their withdrawals. Intuitively, to avoid failure at date 1, the total level of deposits held by the fraction $1-\lambda$ of late depositors at date 1 must be sufficiently large to cover the amount promised to early depositors that cannot be covered by liquidating the investment made by banks $r_1 D_0$.

Figure 2, which represents the individual best response of a given late depositor i given other late depositors' choices, is helpful to characterize the equilibria. By varying the level of δ, figure 2 graphically illustrates how different equilibria configurations may arise.

The return on deposits between dates 1 and 2 depends on whether banks have enough resources available to pay all promised funds to depositors. Hence, bank failure is avoided only when $\rho_2(s)\left(D_1(s) - \frac{r_1 D_0}{1-\lambda}\right) \geq D_1(s)$. The level of deposits that defines the threshold between the failure and no failure regions is given by

$$\frac{1}{1-\frac{1}{\rho_2(s)}}\frac{r_1 D_0}{1-\lambda}, \text{ when } \rho_2(s) > 1,$$

Hence, if $\frac{r_1 D_0}{1-\lambda} \leq D_1(s) < \frac{1}{1-\frac{1}{\rho_2(s)}}\frac{r_1 D_0}{1-\lambda}$, banks fail at date 2, while if $D_1(s) \geq \frac{1}{1-\frac{1}{\rho_2(s)}}\frac{r_1 D_0}{1-\lambda}$, there is no bank failure. Intuitively, when the level of deposit insurance coverage is less than $\frac{1}{1-\frac{1}{\rho_2(s)}}\frac{r_1 D_0}{1-\lambda}$, both the failure equilibrium and the no failure equilibrium are possible. In the failure equilibrium, it is optimal for depositors to leave in the bank an amount of deposits

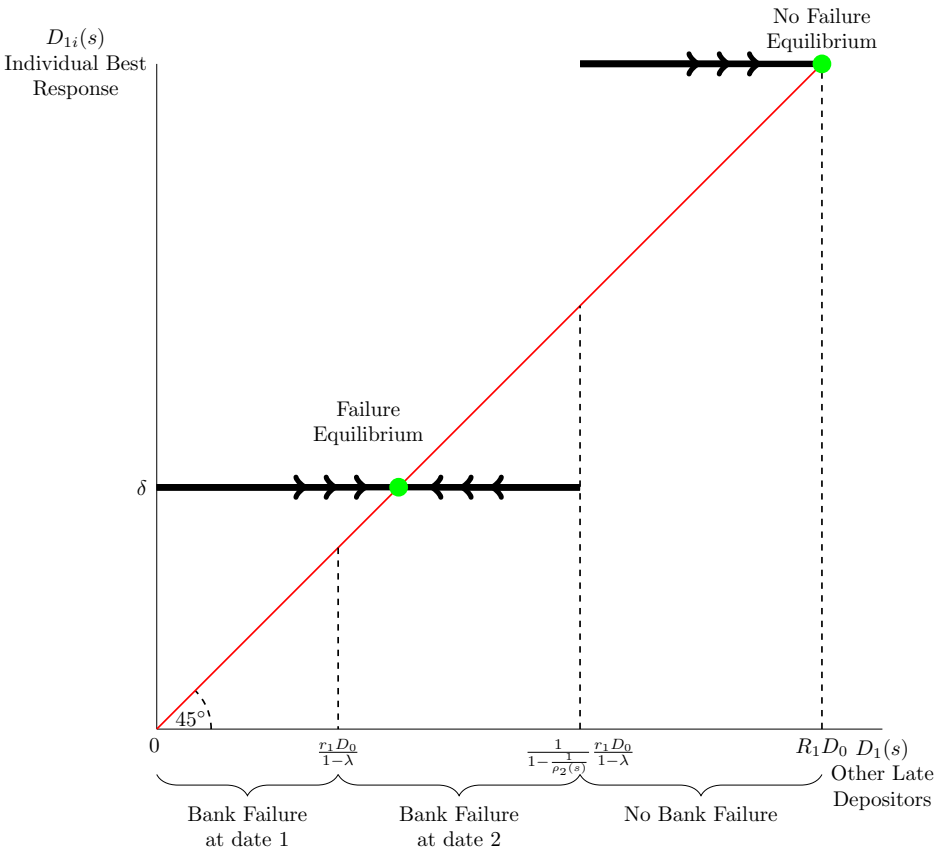

Figure 2: Best response and equilibrium characterization at date 1 for a given realization s

exactly identical to the level of deposit insurance. For instance, the classic run equilibrium of Diamond and Dybvig (1983) corresponds to the case $\delta = 0$. However, a level of deposit insurance higher than $\frac{1}{1-\frac{1}{\rho_2(s)}} \frac{r_1 D_0}{1-\lambda}$ guarantees that $D_1(s) = \delta$ is not an equilibrium, so only the no failure equilibrium is possible. Intuitively, when the level of deposit insurance δ is low, the level of deposits withdrawn from depositors is large enough that banks are not able to satisfy their commitments. However, once the level of deposit insurance is sufficiently high, the mass of funds that remains in the bank is large enough so that it is optimal for all depositors not to withdraw their deposits.

We have thus establish that high enough levels of deposit insurance eliminate the failure equilibrium. However, for this logic to be valid, banks cannot completely insolvent, that is, it must be that $\frac{1}{1-\frac{1}{\rho_2(s)}} \frac{r_1 D_0}{1-\lambda} < R_1 D_0$. Otherwise, only the failure equilibrium exists, independently of the level of deposit insurance coverage. We could describe the situation in which only the failure equilibrium exists as that of a fundamental run. In that case, even if we do not observe a run at date 1, banks will necessarily require government assistance at date 2. Therefore, even when deposit insurance coverage guarantees all deposits, i.e., when $\delta \geq D_0 R_1$, which makes it optimal for depositors to choose $D_1(s) = D_0 R_1$, the depositors' consumption naturally

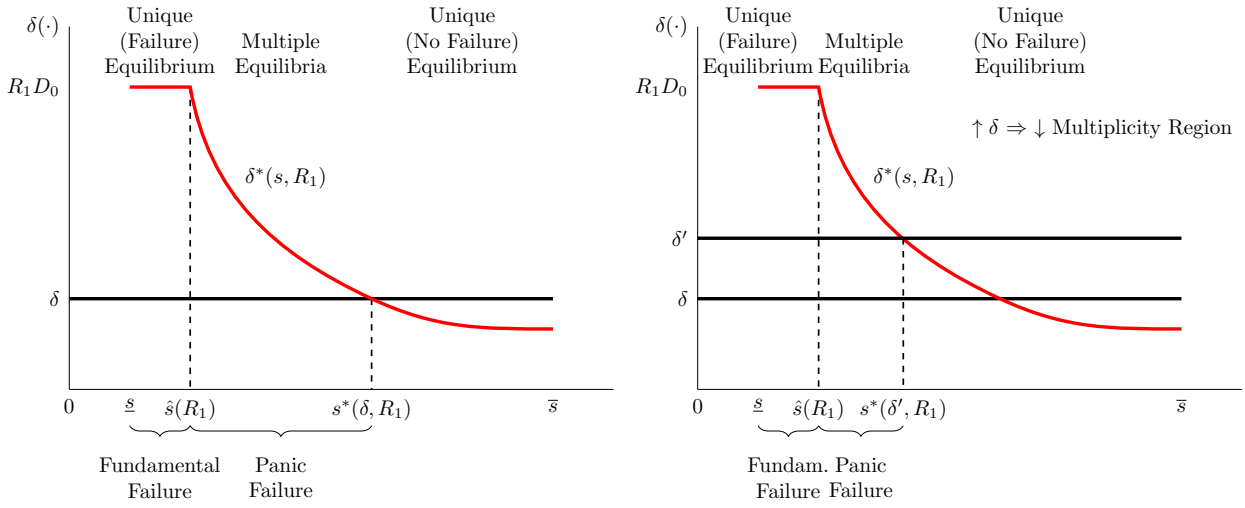

Figure 3: Determination of equilibrium regions and comparative statics on δ

corresponds to the one in the failure equilibrium.

Summing up, for a given realization of s, there are three different configurations of equilibria. Defining $\delta^*(s, R_1)$ as

$$\delta^*(s, R_1) = \frac{1}{1 - \frac{1}{\rho_2(s)}} \frac{r_1 D_0}{1 - \lambda},$$

which is positive as long as $\rho_2(s) > 1$, the possible equilibria configurations for a given realization of s are given by[10]

$$\text{if } \begin{cases} \delta^*(s, R_1) \geq D_0 R_1 \text{ or } \rho_2(s) \leq 1, & \text{Unique (Failure) equilibrium, } D_1 = \min\{\delta, D_0 R_1\} \\ \delta^*(s, R_1) < D_0 R_1, \rho_2(s) > 1, \text{ and } \delta \leq \delta^*(s, R_1), & \text{Multiple equilibria} \\ \delta^*(s, R_1) < D_0 R_1, \rho_2(s) > 1, \text{ and } \delta^*(s, R_1) < \delta, & \text{Unique (No Failure) equilibrium, } D_1 = D_0 R_1 \end{cases}$$

Therefore, at date 1, for a given realization of s, there is a unique equilibrium or multiple equilibria depending on whether $\delta^*(s, R_1)$ is higher or lower than the actual level of deposit insurance δ, as long as it is also less than $D_0 R_1$. When $\rho_2(s)$ is low enough or λ is large enough, no level of deposit insurance is sufficient to eliminate the failure equilibrium. As we show in the appendix, $\frac{\partial \delta^*}{\partial s} \leq 0$ and $\frac{\partial \delta^*}{\partial R_1} > 0$. Intuitively, in good states and when promised deposit rates are lower, a low deposit insurance limit is sufficient to prevent bank failures. Figure 3 illustrates the different regions graphically and shows how they change when δ increases to δ'.

To understand the ex-ante behavior of banks, it is useful to characterize for which realizations of the aggregate state s, the different type of equilibria at date 1 may arise. To do that, we first

[10] Interestingly, the expression for δ^* features a "multiplier" $\frac{1}{1 - \frac{1}{\rho_2(s)}} > 1$. Intuitively, every marginal dollar left inside the banks not only reduces the amount of investments that have to be liquidated, but also earns the extra marginal net return on bank investments. This mechanism amplifies the effect of a given level of coverage. It crucially relies on formulating deposit choices as a continuous variable, which is different from most bank run models with binary (withdraw/not withdraw) deposit decisions.

define two thresholds $\hat{s}(R_1)$ and $s^*(\delta, R_1)$ in the following way:

$$\hat{s}(R_1) : s | D_0 R_1 = \frac{1}{1 - \frac{1}{\rho_2(s)}} \frac{r_1 D_0}{1-\lambda}, \text{ such that } \hat{s}(R_1) \in [\underline{s}, \bar{s}] \tag{4}$$

$$s^*(\delta, R_1) : s | \min\{\delta, D_0 R_1\} = \frac{1}{1 - \frac{1}{\rho_2(s)}} \frac{r_1 D_0}{1-\lambda}, \text{ such that } s^*(\delta, R_1) \in [\underline{s}, \bar{s}] \tag{5}$$

Formally, whenever the solutions for s in equations (4) and (5) lie outside of the interval $[\underline{s}, \bar{s}]$, we force $\hat{s}$ and s^* to take the value of the closest boundary, either $\underline{s}$ or $\bar{s}$. These thresholds allow us to delimit three regions for the type of equilibrium that arises given the realization of the aggregate state:

$$\text{if } \begin{cases} \underline{s} \leq s < \hat{s}(R_1), & \text{Unique (Failure) equilibrium} \\ \hat{s}(R_1) \leq s < s^*(\delta, R_1), & \text{Multiple equilibria} \\ s^*(\delta, R_1) < s \leq \bar{s}, & \text{Unique (No Failure) equilibrium} \end{cases}$$

Figure 4 illustrates the three regions graphically. We show in the appendix that the region of multiplicity decreases with the level of deposit insurance $\frac{\partial s^*}{\partial \delta} \leq 0$. This shows that increasing the level of deposit insurance decreases the region of multiplicity. Both the region of multiplicity and the region with a unique failure equilibrium are increasing in the deposit rate offered by banks, that is, $\frac{\partial s^*}{\partial R_1} \geq 0$ and $\frac{\partial \hat{s}}{\partial R_1} \geq 0$. Note that our formulation accommodates both panic-based failures and fundamental-based failures — see Goldstein (2012) for a recent discussion.

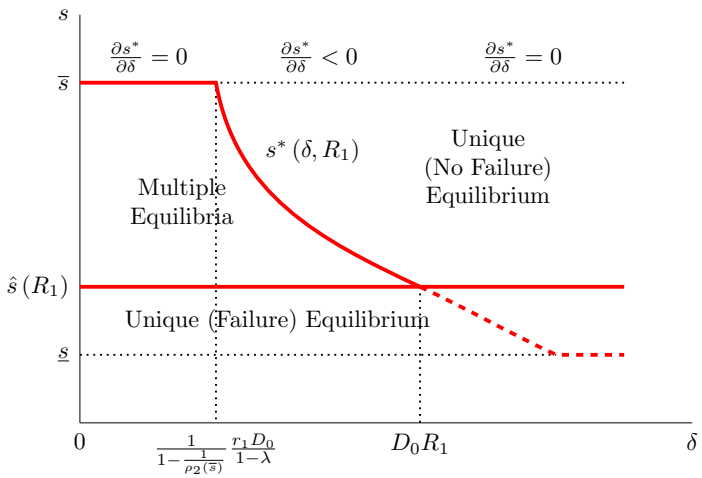

Figure 4: Regions defined by $s^*(\delta, R_1)$ and $\hat{s}(R_1)$

To determine the deposit rate offered by banks at an ex-ante stage, we must take a stand on which equilibrium is actually played for every realization of s. For now, we assume that, given the realization of an aggregate shock in which multiple equilibria are possible, a sunspot coordinates depositors behavior. Hence, for a given realization of s, with probability $\pi \in [0, 1]$

the failure equilibrium occurs and with probability $1-\pi$ the no failure equilibrium occurs.[11] Alternatively, we could have introduced imperfect common knowledge of fundamentals, as in Goldstein and Pauzner (2005), which would allow us to endogenize the probability of bank failure. We show in section 4 that the main insights of this paper extend naturally to that case.

Therefore we can write the unconditional probability of bank failure in this economy, which we denote by $q(\delta, R_1)$, as

$$q(\delta, R_1) = F(\hat{s}(R_1)) + \pi [F(s^*(\delta, R_1)) - F(\hat{s}(R_1))] \tag{6}$$

The unconditional probability of bank failure $q(\cdot)$ inherits the properties of $s^*(\cdot)$ and $\hat{s}(\cdot)$. We show in the appendix that $\frac{\partial q}{\partial \delta} \leq 0$ and $\frac{\partial q}{\partial R_1} \geq 0$. Intuitively, holding the deposit rate constant, higher levels of deposit insurance reduce the likelihood of bank failure in equilibrium, by decreasing the multiple equilibria region. Similarly, holding the level of deposit insurance constant, higher deposit rates offered by banks increase the likelihood of bank failure by reducing the region with a unique no failure equilibrium and by enlarging the region with a unique failure equilibrium. From figure 4, it is easy to establish that $\frac{\partial q}{\partial \delta}$, which plays an important role in our characterization of the optimal policy, is zero for very low and very large values of δ.

Finally, before analyzing banks' choices at date 0, it is helpful to characterize the consumption of late depositors in the different equilibria. In the no failure equilibrium, late depositors consumption at date 2 is given by

$$C_{2N}(s, \delta, R_1) = \frac{\rho_2(s) D_0 (1 - \lambda R_1)}{1 - \lambda} + Y_2(s) \tag{7}$$

No taxes need to be raised when banks do not fail. In the failure equilibrium, we decompose the equilibrium consumption of late depositors into two components. Late depositors consumption at date 2 is given by

$$C_{2R}(s, \delta, R_1) = \tilde{C}_{2R}(s, \delta, R_1) - T_2(s, \delta, R_1), \tag{8}$$

where we use the index R for the bank failure cases, standing for "run equilibrium". The consumption of a late depositor at date 2 in a failure equilibrium before taxes, denoted by $\tilde{C}_{2R}(s, \delta, R_1)$, is given, as we show in the appendix, by

$$\tilde{C}_{2R}(s, \delta, R_1) = \left[\delta + \frac{D_0 (1 - \lambda R_1)}{1 - \lambda}\right] (1 - \mathbb{I}_1) + D_0 R_1 \mathbb{I}_1 + Y_2(s),$$

where we use $\mathbb{I}_1$ to denote an indicator that corresponds to the region in which banks are able to satisfy all date 1 withdrawals, formally:[12]

$$\mathbb{I}_1 \equiv \mathbb{I}\left[\delta \geq \frac{r_1 D_0}{1 - \lambda}\right]$$

[11] We can easily allow for a value of π contingent on the aggregate state s, $\pi(s)$.

[12] We use an indicator instead of defining thresholds for this region to simplify the exposition. This approach is valid because the consumption of late depositors is continuous when $\delta = \frac{r_1 D_0}{1-\lambda}$.

We denote the by $T_2(s, \delta, R_1)$ the funds that need to be raised to pay for deposit insurance. In the failure equilibrium, the deposit insurance authority must raise the minimum between δ or the total level of deposits from each late depositor since, as we have shown, it is optimal for all late depositors to keep δ deposits in the bank at date 1. Hence, under our assumption that the deposit insurance authority can only recover a fraction χ of funds from banks in case of bank failure and that the net marginal cost of public funds is κ, whenever the failure equilibrium occurs, the level of funds to be raised from a given late depositor is determined by

$$T_2(s, \delta, R_1) = (1+\kappa)\left[\min\{\delta, D_0 R_1\} - \chi\rho_2(s)\left(\min\{\delta, D_0 R_1\} - \frac{r_1 D_0}{1-\lambda}\right)\mathbb{I}_1\right]$$

It is easy to show that $T_2(s, \delta, R_1) \geq 0$.[13] We also show that the amount of funds that must be raised to pay for deposit insurance in a failure equilibrium increases with R_1 but, more surprisingly, it can increase or decrease with δ. Formally, we show that $\frac{\partial T_2}{\partial R_1} > 0$ but $\frac{\partial T_2}{\partial \delta} \gtreqless 0$. It can be the case that increasing the level of coverage becomes self financing, since the returns on the banks investments yields $\rho_2(s) > 1$ units of output. Note that $\frac{\partial T_2(s,\delta,R_1)}{\partial s} < 0$, since less public funds are needed when banks returns are higher.

Consolidating both terms, the total consumption for a late depositor at date 1, in the relevant interior case $\delta < R_1 D_0$, can be thus written as

$$C_{2R}(s, \delta, R_1) = \frac{D_0(1-\lambda R_1)}{1-\lambda} - \kappa\delta + \left((1+\kappa)\chi\rho_2(s) - 1\right)\left(\delta - \frac{r_1 D_0}{1-\lambda}\right)\mathbb{I}_1 + Y_2(s)$$

In the appendix, we prove that, for a given realization of the aggregate state, the expected consumption of late depositors is higher in the no failure equilibrium, that is, $C_{2N} - C_{2R} > 0$.

From now on, to ease the notation, we omit the arguments of many different functions, unless we want to make a special emphasis on the dependence of some variables.

Banks Since we have assumed that banks are perfectly competitive, they offer at date 0 a rate of return on deposits R_1 which maximizes the ex-ante welfare of depositors. Banks are aware of the possibility of bank failure and internalize how the choice of R_1 affects the likelihood and severity of bank failure. On the contrary, because they are small, banks fail to internalize how their actions affect the level of taxes T_2 that must be raised in case of bank failure.

For a given level of deposit insurance δ, depositors indirect utility from an ex-ante viewpoint

[13] An alternative timing assumption in which funds have to be raised first and then the unwinding of banks assets occurs corresponds to $T_2(s, \delta, R_1) = (1+\kappa)\delta - \chi\rho_2(s)\left(\delta - \frac{r_1 D_0}{1-\lambda}\right)(1-\mathbb{I}_1)$. The differences between both formulations are minimal. See FDIC (1998) for how both procedures have been used in practice over time.

can be written, as a function of R_1, as follows

$$J(R_1;\delta) = \lambda U(R_1 D_0 + Y_1) + (1-\lambda)\left[\begin{array}{c}\int_{\underline{s}}^{\hat{s}(R_1)} U(C_{2R}(s))\,dF(s) + \int_{\hat{s}(R_1)}^{s^*(\delta,R_1)} (\pi U(C_{2R}(s)) + (1-\pi)U(C_{2N}(s)))\,dF(s) \\ + \int_{s^*(\delta,R_1)}^{\bar{s}} U(C_{2N}(s))\,dF(s)\end{array}\right], \tag{9}$$

where $C_{2N}(s)$ and $C_{2R}(s)$ are respectively defined in equations (7) and (8).

Hence, banks choose R_1 to solve

$$R_1^*(\delta) = \arg\max_{R_1} J(\delta, R_1)|_{T_2},$$

where $J(\delta, R_1)|_{T_2}$ corresponds to equation (9), taking T_2 as given. For a given level of deposit insurance δ, under appropriate regularity conditions, $R_1^*(\delta)$ is given by the solution to $\left.\frac{\partial J}{\partial R_1}\right|_{T_2} = 0$, where

$$\begin{aligned}\left.\frac{\partial J}{\partial R_1}\right|_{T_2} &= \lambda U'(D_0 R_1 + Y_1) D_0 + (1-\lambda)\int_{\underline{s}}^{\hat{s}} U'(C_{2R}(s))\frac{\partial \tilde{C}_{2R}(s)}{\partial R_1} dF(s) \\ &+ (1-\lambda)\left[\int_{\hat{s}}^{s^*}\left(\begin{array}{c}\pi U'(C_{2R}(s))\frac{\partial \tilde{C}_{2R}(s)}{\partial R_1} \\ + (1-\pi)U'(C_{2N}(s))\frac{dC_{2N}(s)}{dR_1}\end{array}\right) dF(s) + \int_{s^*}^{\bar{s}} U'(C_{2N}(s))\frac{dC_{2N}(s)}{dR_1} dF(s)\right] \\ &+ (1-\lambda)(1-\pi)\left[U(C_{2R}(\hat{s})) - U(C_{2N}(\hat{s}))\right]\frac{\partial \hat{s}}{\partial R_1} f(\hat{s}) \\ &+ (1-\lambda)\pi\left[U(C_{2R}(s^*)) - U(C_{2N}(s^*))\right]\frac{\partial s^*}{\partial R_1} f(s^*)\end{aligned} \tag{10}$$

The date 2 derivatives of late depositors' consumption are given by

$$\frac{dC_{2N}}{dR_1} = -\rho_2(s)\frac{\lambda}{1-\lambda} D_0 < 0 \quad \text{and} \quad \frac{\partial \tilde{C}_{2R}}{\partial R_1} = \left[-\frac{\lambda}{1-\lambda}(1-\mathbb{I}_1) + \mathbb{I}_1\right] D_0,$$

and, as shown above, both $\frac{\partial \hat{s}(R_1)}{\partial R_1}$ and $\frac{\partial s^*(\delta,R_1)}{\partial R_1}$ are positive.

The choice of R_1 determines the optimal degree of risk sharing between early and late types, accounting for the level of aggregate uncertainty and incorporating the costs associated with bank failure. Overall, banks internalize that varying R_1 changes the consumption of depositors for given failure and no failure states (intensive margin terms) and the likelihood of experiencing a bank failure (extensive margin terms). Importantly, banks do not take into account how their choice of R_1 affects the need to raise resources through taxation to pay for deposit insurance.

An increase in R_1 always increases the consumption of early depositors and, in general, reduces the consumption of late depositors: this is captured by the negative signs of $\frac{dC_{2N}}{dR_1}$ and $\frac{\partial \tilde{C}_{2R}}{\partial R_1}$. Only when $\mathbb{I}_1 = 1$, banks perceive that increasing R_1 benefits both early and late depositors

at the margin. Banks take into account that offering a high deposit rate makes bank failures more likely. This is captured by the positive sign of $\frac{\partial \hat{s}}{\partial R_1}$ and $\frac{\partial s^*}{\partial R_1}$, which combined with the sign of $U(C_{2R}) - U(C_{2N})$, which we know to be negative, makes increasing R_1 less desirable. When $\pi \to 0$, and $\hat{s} = \underline{s}$, equation (8) corresponds exactly to the optimal arrangement that equalizes marginal rates of substitution across types with the expected marginal rate of transformation shaped by $\rho_2(s)$. In that case, banks set R_1 exclusively to provide insurance between early and late types.

Although, theoretically, the sign of $\frac{dR_1^*}{d\delta}$ is unclear, due to conflicting income effects and direct effects on the size of the failure/non-failure regions, R_1 increases with δ in most situations, that is, $\frac{dR_1^*}{d\delta} > 0$ — we find this behavior in the numerical example described in the appendix. Intuitively, since the consumption of late depositors increases with the level of coverage and the likelihood of failure is smaller, we expect that banks optimally decide to offer higher deposit rates when deposit insurance coverage is more generous. This result is a form of moral hazard by banks. In section 4, banks also choose the composition of their investment, which makes the effect of banks behavioral responses on welfare more salient.

Finally, it is clear that $J(R_1; \delta)$ is continuous in R_1, although it may be non-differentiable at a finite number of points. For the characterization of equation (10) to be valid, we work under the assumption that R_1^* is found at an interior optimum. Since by adding some observable noise we can make $J(R_1; \delta)$ everywhere differentiable, this assumption does not entail great loss of generality.

3 Normative analysis

After characterizing the behavior of this economy for a given level of deposit insurance δ, we now study how social welfare varies with δ and characterize the socially optimal level of deposit insurance δ^*. We first analyze the case in which no ex-ante policies are available and then extend our analysis to the more realistic case in which ex-ante corrective policies can be used.

3.1 Optimal deposit insurance δ^*

First, we study how welfare changes with the level of deposit insurance. Social welfare in this economy is given by the ex-ante expected utility of depositors. We denote social welfare, written

as a function of the level of deposit insurance, by $W(\delta)$. Formally, $W(\delta)$ is given by

$$W(\delta) = \lambda U\left(R_1^*(\delta) D_0 + Y_1\right) + \tag{11}$$
$$+ (1-\lambda)\left[\begin{array}{c} \int_{\underline{s}}^{\hat{s}\left(R_1^*(\delta)\right)} U\left(C_{2R}\left(s,\delta,R_1^*(\delta)\right)\right) dF(s) \\ + \int_{\hat{s}\left(R_1^*(\delta)\right)}^{s^*\left(\delta,R_1^*(\delta)\right)} \left(\pi U\left(C_{2R}\left(s,\delta,R_1^*(\delta)\right)\right) + (1-\pi) U\left(C_{2N}\left(s,R_1^*(\delta)\right)\right)\right) dF(s) \\ + \int_{s^*\left(\delta,R_1^*(\delta)\right)}^{\overline{s}} U\left(C_{2N}\left(s,R_1^*(\delta)\right)\right) dF(s) \end{array}\right],$$

where $C_{2N}\left(s,\delta,R_1^*(\delta)\right)$ and $C_{2R}\left(s,\delta,R_1^*(\delta)\right)$ are respectively defined in equations (7) and (8) and $R_1^*(\delta)$ is given by the solution to equation (10). The first term of $W(\delta)$ is the expected utility of early depositors. The second term, in brackets, is the expected utility of late depositors. It accounts for the equilibria that will occur for the different realizations of the aggregate state.

Proposition 1 presents the first main result of this paper.

Proposition 1. (Marginal effect on welfare of varying the level of deposit insurance δ) *The change in welfare induced by a marginal change in the level of deposit insurance $\frac{dW}{d\delta}$ is given by*

$$\frac{\frac{dW}{d\delta}}{1-\lambda} = \left(U\left(C_{2R}\left(s^*\right)\right) - U\left(C_{2N}\left(s^*\right)\right)\right)\frac{\partial q}{\partial \delta} \tag{12}$$
$$+ q\mathbb{E}_R\left[U'\left(C_{2R}(s)\right)\left(\kappa + \left(1-(1+\kappa)\chi\rho_2(s)\right)\mathbb{I}_1 + \frac{\partial T_2}{\partial R_1}\frac{dR_1}{d\delta}\right)\right],$$

where $\mathbb{E}_R[\cdot]$ stands for a conditional expectation over bank failure states and, as defined above, q denotes the unconditional probability of bank failure, $\frac{\partial q(\delta)}{\partial \delta} = \pi f\left(s^*(\delta)\right)\frac{\partial s^*(\delta)}{\partial \delta}$ and $\mathbb{I}_1 = \mathbb{I}\left(\delta \geq \frac{r_1 D_0}{1-\lambda}\right)$.

Proposition 1 characterizes the effect on welfare of a marginal change in the level of deposit insurance. The first line of equation (12) captures the marginal benefit of increasing deposit insurance by a dollar, while its second line captures the marginal cost of doing so. On the one hand, a higher level of deposit insurance decreases the likelihood of bank failure by $\frac{\partial q}{\partial \delta} < 0$. This reduction creates a welfare gain given by the wedge in depositors' utility between the failure and no failure states $U\left(C_{2R}\left(s^*\right)\right) - U\left(C_{2N}\left(s^*\right)\right)$, which we show must be negative. Hence, we can express the benefit of increasing the deposit insurance limit as

$$\underbrace{\underbrace{\left(U\left(C_{2R}\left(s^*\right)\right) - U\left(C_{2N}\left(s^*\right)\right)\right)}_{\text{Utility Drop}} \underbrace{\frac{\partial q(\delta)}{\partial \delta}}_{\text{Change in Failure Probability}}}_{\text{Benefit of DI}}$$

On the other hand, a higher level of deposit insurance changes the consumption of late depositors in case of bank failure by

$$\frac{\partial C_{2R}}{\partial \delta} - \frac{\partial T_2}{\partial R_1}\frac{dR_1}{d\delta} = \underbrace{\underbrace{-\kappa}_{\text{Cost of Public Funds}} \underbrace{-\left(1-(1+\kappa)\chi\rho_2(s)\right)\mathbb{I}_1}_{\text{Illiquidity/Insolvency}} \underbrace{-\frac{\partial T_2}{\partial R_1}\frac{dR_1}{d\delta}}_{\text{Fiscal Externality}}}_{\text{Cost of DI}} \tag{13}$$

The first term of $\frac{\partial C_{2R}}{\partial \delta}$ is the net marginal cost of public funds associated with a unit increase in the level of deposit insurance coverage. Intuitively, a higher δ increases the transfers towards depositors, which have a net fiscal unit cost of κ.

The second term of $\frac{\partial C_{2R}}{\partial \delta}$ is the net social value of leaving one more dollar of deposits inside the banks and it captures whether banks are simply illiquid or insolvent. This term is nonzero whenever banks do not fully liquidate their investments at date 1, that is, when $\mathbb{I}_1 = 1$, and it captures whether deposit insurance keeps unprofitable banks (inefficiently) functioning or (efficiently) supports insolvent but profitable investments. The illiquidity/insolvency term corresponds to the difference between the (unit) gain from liquidating a unit of investment at date 1 and the social returns obtained by leaving that extra unit inside the banks, which corresponds to $\rho_2(s)$, corrected by the liquidation loss χ and marginal fiscal saving κ.

The first two terms combined can take the value $-\kappa$ when $\mathbb{I}_1 = 0$ or $-(1+\kappa)(1-\chi\rho_2(s))$ when $\mathbb{I}_1 = 1$. Hence, for this term to be negative at a given state, it must be that deposit insurance is preventing inefficient investments to be liquidated, which occurs when $1 > \chi\rho_2(s)$. Note that, even when $\kappa = 0$ and it is free to raise public funds $\frac{\partial C_{2R}}{\partial \delta}$ is non-zero and equals $1 - \chi\rho_2(s)$.[14] All these effects are smoothed out, since the relevant variable is the ex-ante expectation of the marginal effects.

The third and final term corresponds to the impact of the distortions on banks' behavior induced by the change in level of deposit insurance. We have shown that $\frac{\partial T_2}{\partial R_1}$ is always positive and argued that $\frac{dR_1}{d\delta}$ is also positive, so this third term in equation (13) increases the marginal cost of increasing the deposit insurance limit. Because it affects directly the funds that need to be raised by the government, we refer to it as a fiscal externality. Depositors value any change in consumption at their marginal utility $U'(C_{2R})$. We further discuss this term in our first remark of this section.

The derivation of equation (12) repeatedly exploits the fact that banks choose the value of R_1 to provide insurance across types optimally, while taking into account how that may change the likelihood of bank failure. Moreover, it provides a simple test for whether to increase or decrease the level of deposit insurance.

Under appropriate regularity conditions, the optimal level of deposit insurance must be a solution to the equation $\frac{dW}{d\delta} = 0$. Proposition 2 characterizes the optimal level of deposit insurance at an interior optimum.

Proposition 2. (Optimal deposit insurance) *The optimal level of deposit insurance δ^* is characterized by*

$$\delta^* = \frac{\varepsilon_\delta^q \left(U\left(C_{2R}(s^*)\right) - U\left(C_{2N}(s^*)\right)\right)}{q\mathbb{E}_R\left[U'\left(C_{2R}(s)\right)\left(\kappa + \left(1-(1+\kappa)\chi\rho_2(s)\right)\mathbb{I}_1 + \frac{\partial T_2}{\partial R_1}\frac{dR_1}{d\delta}\right)\right]}, \tag{14}$$

[14] When $1 > \chi\rho_2(s)$, competitive bank managers could find optimal to liquidate all investments and return all proceeds to depositors. This possibility is ruled out because contracts are non-contingent.

where $\mathbb{E}_R[\cdot]$ *stands for a conditional expectation over bank failure states and, as defined above,* q *denotes the unconditional probability of bank failure,* $\varepsilon_\delta^q = \frac{\partial q(\delta)}{\partial \log(\delta)}$ *denotes the change in the likelihood of bank failure induced by a percent change in the level of deposit insurance and* $\mathbb{I}_1 = \mathbb{I}\left(\delta \geq \frac{r_1 D_0}{1-\lambda}\right)$.

The optimal level of deposit insurance trades off the welfare gains from reducing the likelihood of bank failure (numerator) with the fiscal cost associated with deposit insurance (denominator). At an interior optimum, we expect both numerator and denominator to be strictly positive, so that $\delta^* > 0$. Intuitively, a high value for δ^* is optimal when ε_δ^q and $U(C_{2R}(s^*)) - U(C_{2N}(s^*))$ are large in magnitude. If the reduction in the probability of bank failure is large at the same time that the welfare loss caused at the margin by a bank failure is also large, it is optimal to have a large level of deposit insurance. A low value for δ^* is optimal when the probability of actually paying for deposit insurance is high, at the same time that the net marginal cost of public funds κ is high, the recovery rate for the government out banks investments is low and the sensitivity of the deposit rate offered banks with respect to the coverage level is high.

Equation (14) encapsulates all the relevant tradeoffs that optimally determine the deposit insurance limit. Importantly, it characterizes δ^* as a function of a few sufficient statistics, which can be potentially be recovered from actual data, a fact that we exploit in section 5. However, as it is common in optimal policy exercises, δ^* cannot be written as a function of primitives, since all right hand side variables in equation (14) are endogenous.[15]

We would like to emphasize four implications of our optimal policy formulas in propositions 1 and 2.

Remark 1. **Banks behavioral responses (often referred to as moral hazard) only affect social welfare directly through the fiscal externality term.** It is true that we expect banks to quote higher deposit rates when the level of deposit insurance is higher, since they know that depositors consumption is partially shielded by the existence of deposit insurance. However, because banks are competitive and maximize depositors welfare, only the fiscal consequences of their change in behavior, which materializes when the fiscal authority actually has to pay for deposit insurance, matters. This result remains valid even when banks have an endogenous choice of investment — see section 4. This is an important takeaway of this paper and crucially relies on the fact that banks are perfectly competitive.

Moral hazard on the banks side only affects directly the optimal deposit insurance limit through its fiscal cost, although it can indirectly affect the level of gains from reducing bank failures (in the numerator of (14)), the region in which deposit insurance is paid (in the denominator of (14)) and the value attached to a dollar in the different states (captured in

[15] This logic is similar to classic characterizations of optimal taxes. For instance, the demand elasticities in optimal Ramsey commodity taxes are a function of demand elasticities, which are endogenous to the level of taxes — see Atkinson and Stiglitz (1980).

depositors marginal utility). An interesting benchmark is $\chi = 0$. In that case, because $\frac{\partial T_2}{\partial R_1} = 0$, the term associated with fiscal externality goes away, so any change in banks' behavior caused by varying the level of δ exclusively affect caused by the change in behavior of banks goes away.

Remark 2. **(Diamond and Dybvig (1983) revisited) When deposit insurance involves no payments in equilibrium, the optimal policy fully insures deposits.** In an environment without aggregate risk, Diamond and Dybvig (1983) show that it is optimal to provide unlimited deposit insurance to avoid the bank failure equilibrium.[16] In their model, unlimited deposit insurance eliminates bank failures altogether but, more importantly, deposit insurance never has to be paid in equilibrium. We can intuitively understand their results by looking at proposition 1: because public funds are never raised to pay for deposit insurance in equilibrium, the second line in equation (12) is zero, since $q = 0$, but the first line will be positive at some value, which implies that the optimal level of insurance is the highest possible one.

Remark 3. **Deposit insurance may become self financing even when it has to be paid in some states.** This occurs when $(1+\kappa)\chi\rho_2(s) > 1$. The left hand side of this expression captures the benefit of leaving an extra dollar of deposits inside a bank in a bank failure situation. It involves the fiscal savings $1+\kappa$, the cost of liquidation χ and, crucially, the return on assets $\rho_2(s)$. When $\rho_2(s)$ is sufficiently large, we capture the possibility that keeping a dollar inside the bank and not liquidating its projects turns out to generate positive returns for the government. This term captures the extra gain from avoiding bank failures in situations in which banks are illiquid but are able to yield positive returns if their investments are not liquidated. When $(1+\kappa)\chi\rho_2(s) < 1$, this term corresponds to the extra resource cost caused by deposit insurance by increasing the level of deposits in insolvent banks which ought to be liquidated.

Remark 4. **Small levels of deposit insurance decrease welfare.** Formally, we show that

$$\left.\frac{dW}{d\delta}\right|_{\delta=0} < 0$$

Intuitively, low levels of deposit insurance are ineffective to prevent bank failures at all. However, when a bank fails, the government incurs the fiscal cost associated with paying for deposit insurance. Only when the deposit insurance limit is sufficiently large, the benefits from insuring deposits materialize. This result shows that the planning problem — illustrated in figure 5, and described in more detail in the numerical example in the appendix — suffers from a very specific form of non-convexity.

Figure 5 illustrates how social welfare varies with the level of deposit insurance. The value that maximizes $W(\delta)$ is δ^*, which we have characterized in equation (14). See the appendix for

[16] More precisely, Diamond and Dybvig (1983) propose two solutions to avoid costly bank runs: suspension of convertibility and deposit insurance. We are exclusively focusing on the role of deposit insurance.

a numerical example.[17]

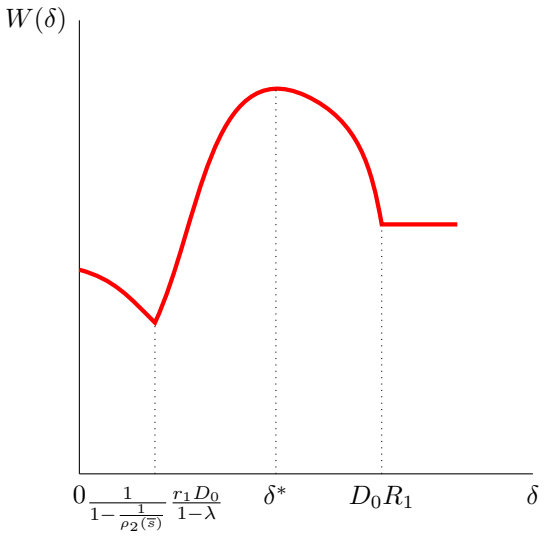

Figure 5: Social welfare $W(\delta)$

Finally, although we emphasize the formula for δ^* because of its simplicity, our characterization of $\frac{dW}{d\delta}$ might be more relevant in practical terms. Taking into account the possible non-convexity in social welfare that we have just discussed, equation (12) provides in general a simple test for whether it is optimal to increase or decrease the level of deposit insurance. Although it is derived for marginal changes, it is easy to evaluate the change in welfare caused by any discrete jump in the level of deposit insurance by integrating over the values of $\frac{dW}{d\delta}$, using the fundamental theorem of calculus. Formally, when δ moves from δ_0 to δ_1, we can write the welfare change as

$$W(\delta_1) - W(\delta_0) = \int_{\delta_0}^{\delta_1} \frac{dW}{d\delta}(\tilde{\delta})\, d\tilde{\delta},$$

where $\frac{dW}{d\delta}(\cdot)$ is determined in proposition 1.

3.2 Ex-ante corrective policies

Up to now, we have assumed that banks can freely choose the deposit rate that they offer to their depositors. However, it is natural to allow the policymaker to jointly determine the optimal level of deposit insurance along with a set of ex-ante policies that modify the behavior of banks at date 0. It should be obvious that allowing the deposit insurance authority to affect the ex-ante behavior of depositors must improve welfare. We first characterize the set of constrained

[17] It is not easy to compare the values of $W(0)$ and $W(\infty)$, so it is not possible in general to easily rank systems with full coverage of deposits versus those without deposit insurance.

efficient policies and then discuss possible decentralizations, for instance, imposing deposit rate ceilings or setting a deposit insurance premium.

The planner now chooses jointly the level of δ^* and the deposit rate offered to households. The optimal choice of R_1 is again characterized by the solution to $\frac{\partial J}{\partial R_1} = 0$, with the caveat that now the planner fully internalizes the effect of changing R_1 on the required level of fiscal revenues T_{2R}, that is, $\frac{dC_{2R}}{dR_1}$ substitutes $\frac{\partial \tilde{C}_{2R}}{\partial R_1}$ in equation (10). This policy is akin to regulating the deposit rate offered by banks covered by deposit insurance guarantees. Deposit rate regulation has been commonly used in practice, in particular before the financial deregulation wave that starts in the 1980's.

We can then characterize the level of δ^* as follows.

Proposition 3. (Optimal deposit insurance with ex-ante corrective policies) *a) When the planner can control the deposit rate offered by banks, the optimal level of deposit insurance δ^* is characterized by*

$$\delta^* = \frac{\varepsilon_\delta^q \left(U\left(C_{2R}\left(s^*\right)\right) - U\left(C_{2N}\left(s^*\right)\right)\right)}{q\mathbb{E}_R\left[U'\left(C_{2R}\left(s\right)\right)\left(\kappa + \left(1-\left(1+\kappa\right)\chi\rho_2\left(s\right)\right)\mathbb{I}_1\right)\right]}, \tag{15}$$

b) The optimal corrective policy modifies the optimal choice of deposit rates by banks introducing a wedge in their deposit rate decision given by

$$\tau_{R_1} \equiv q\mathbb{E}_R\left[\left(1-\lambda\right)U'\left(C_{2R}\right)\frac{\partial T_2}{\partial R_1}\right] \tag{16}$$

When compared to proposition 2, the optimal deposit insurance formula does not contain the fiscal externality term. By introducing a second instrument, only the marginal cost of public funds, corrected by the liquidation wedge, matter to determine δ^*. The optimal ex-ante policy forces banks to internalize how the choice of deposit rates affects the level of available resources in case of bank failure. To provide further intuition, we show in the appendix that $\left(1-\lambda\right)\frac{\partial T_2}{\partial R_1} = \left(1+\kappa\right)\chi\rho_2\left(s\right)D_0\mathbb{I}_1 \geq 0$. Hence, we can write the wedge on the choice of deposit rates as

$$\tau_{R_1} \equiv \left(1+\kappa\right)\chi q\mathbb{E}_R\left[U'\left(C_{2R}\right)\rho_2\left(s\right)D_0\mathbb{I}_1\right]$$

Intuitively, banks should internalize that a higher deposit rate entails the need to liquidate a higher number of investments to pay for the deposits of early types, which means that fewer resources are available in bank failure states. The marginal loss of resources is precisely given by $\rho_2\left(s\right)D_0$, valued at the marginal utility of late depositors $U'\left(C_{2R}\right)$. Naturally, this marginal loss only materializes in those states in which there are some funds left.

It is worth emphasizing that when the recovery rate banks investments is zero, that is when $\chi = 0$, the private choice of R_1 and the one chosen by the planner fully coincide: in that case, $\frac{\partial T_2}{\partial R_1} = 0$. Having access to ex-ante corrective instruments is irrelevant in that case. Intuitively, the only component of social welfare non-internalized by banks depends on the

level of funds available in bank failure states, which happen to be zero when all proceeds from banks' investment are lost.

In general, the implementation of the optimal ex-ante corrective policy is not unique, although in this particular case a single instrument affecting the choice of deposit rate is sufficient. Because the funds used to pay for deposit insurance are raised through distortionary taxation, any Pigovian corrective policy in which the deposit insurance authority raises revenue may generate a "double-dividend".[18] That is, a policy that corrects the ex-ante behavior of banks at the same time that reduces the need for raising fiscal revenue when required can improve welfare in two different margins. This argument provides support for an implementation of the optimal corrective policy through a deposit insurance fund financed with a deposit insurance premia, provided that the returns of those funds are comparable to return of banks. In general, our analysis highlights the distinction between the corrective role of ex-ante policies versus its revenue raising role. This is an argument often blurred in previous discussions of these issues. In the next section, we show how our framework has sharp predictions for the optimal deposit insurance premium in a more general model.

Finally, note that we have characterized two extreme situations. In one, there is no ex-ante regulation, so banks choose freely their deposit rate. In another one, there is perfectly targeted regulation. Any restriction on the set of feasible instruments available to the policymaker — which may arise from informational frictions about banks characteristics, from which we abstract in this paper — should deliver an intermediate outcome.

4 Extensions

The basic framework in which we derive the main insights of the paper contains a number of restrictions. The goal of this section is to show that our results are robust to multiple natural generalizations. To ease the exposition, we study every extension separately, omitting regularity conditions. We focus on the characterization of the optimal level of deposit insurance, relegating the characterization of marginal changes in the level of deposit insurance to the appendix.

4.1 General portfolio and investment decisions

In our basic framework, neither depositors nor banks had portfolio decisions. We now relax that assumption and show that the main characterization and the insights from the basic model remain robust.

On the one hand, depositors continue to be ex-ante identical and deposit a fixed amount D_0

[18] See Goulder (1995) for a discussion of classic double-dividend arguments in the context of environmental regulation.

of resources. However, now they have a consumption-savings decision at date 0 and a portfolio decision among K securities. Depositors have access to $k = 1, 2, \ldots, K$ assets, with returns $\rho_{1k}(s)$ at date 1 in state s for early depositors and returns $\rho_{2k}(s)$ at date 2 in state s for late depositors. Hence, the resources of early and late depositors are respectively given by $Y_1(s) = \sum_k \rho_{1k}(s) y_k$ and $Y_2(s) = \sum_k \rho_{2k}(s) y_k$. The budget constraint of depositors at date 0 is given by

$$\sum_k y_k + D_0 + C_0 = Y_0, \tag{17}$$

where Y_0, which denotes the initial wealth of depositors, and D_0 are primitives of the model. Subject to equation (17), the ex-ante utility of depositors now corresponds to

$$\max_{y_k, D_{1i}(s)} U(C_0) + \mathbb{E}_s \left[\lambda U(C_{1i}(s)) + (1-\lambda) U(C_{2i}(s))\right]$$

On the other hand, banks have access to $j = 1, 2, \ldots, J$ investment opportunities, which offer a return $\rho_{2j}(s)$ at date 2 and can be liquidated at date 1 receiving a one-for-one return. Hence, at date 0, banks must choose weights ψ_j for every investment opportunity such that $\sum_j \psi_j = 1$. We assume that banks liquidate an equal fraction φ of every type of investment at date 1. This is a particularly tractable formulation to introduce multiple investment opportunities. Our results naturally extend to the case in which different investments have different liquidation rates at date 1 and banks have the choice of liquidating different investments in different proportions.

Given our assumptions, we show that the level of available funds at date 2 — the counterpart to equation (3) in our basic framework — is given by

$$\sum_j \rho_{2j}(s) \psi_j \left(D_1(s) - \frac{r_1 D_0}{1-\lambda}\right), \tag{18}$$

where the derivation of (18) used the fact that $\varphi = \frac{\Delta(s)}{\sum \psi_j D_0} = \frac{\Delta(s)}{D_0}$. Given date 0 choices, the characterization of the equilibrium remains identical, with δ^* now given by $\delta^* = \frac{1}{1 - \frac{1}{\sum_j \rho_{2j}(s)\psi_j}} \frac{r_1 D_0}{1-\lambda}$. The level of funds to be raised in a bank failure equilibrium, which is relevant for the determination of the optimal policy, is now given by

$$T_2(s, \delta, R_1, D_0, \psi_j) = (1+\kappa)\left[\min\{\delta, D_0 R_1\} - \sum_j \rho_{2j}(s) \psi_j \left(\min\{\delta, D_0 R_1\} - \frac{r_1 D_0}{1-\lambda}\right)\right]$$

We characterize in the appendix the optimal choices of y_k and ψ_j by depositors and banks and focus directly on the optimal policy results.

Proposition 4. (Optimal DI with general portfolio and investment decisions) *a) The optimal level of deposit insurance δ^*, without ex-ante regulation, is characterized by*

$$\delta^* = \frac{\varepsilon_\delta^q \left(U(C_{2R}(s^*)) - U(C_{2N}(s^*))\right)}{q \mathbb{E}_R \left[U'(C_{2R}(s)) \left(\kappa + (1 - (1+\kappa)\chi \rho_2(s)) \mathbb{I}_1 + \frac{\partial T_2}{\partial R_1}\frac{dR_1}{d\delta} + \sum_j \frac{\partial T_2}{\partial \psi_j}\frac{d\psi_j}{d\delta}\right)\right]},$$

where $\mathbb{E}_R[\cdot]$ *stands for a conditional expectation over bank failure states and* $\varepsilon_\delta^q = \frac{\partial q(\delta)}{\partial \log \delta}$ *denotes the change in the likelihood of bank failure induced by a one percent change in the level of deposit insurance.*

b) The optimal level of deposit insurance δ^**, when the policymaker has access to ex-ante regulation, is characterized by*

$$\delta^* = \frac{\varepsilon_\delta^q \left(U\left(C_{2R}\left(s^*\right)\right) - U\left(C_{2N}\left(s^*\right)\right)\right)}{q\mathbb{E}_R\left[U'\left(C_{2R}\left(s\right)\right)\left(\kappa + \left(1-\left(1+\kappa\right)\chi\rho_2\left(s\right)\right)\mathbb{I}_1\right)\right]} \tag{19}$$

This expression is identical to the one in proposition 4.

Proposition 4 is a natural extension to our results in the more stylized model. Importantly, it shows that introducing a consumption-savings and portfolio choices for depositors does not modify the set of sufficient statistics already identified. However, allowing banks to make investment choices introduces a new term into the optimal policy formula. The new fiscal externality term, which captures the direct effects of banks behavioral responses, is now given by

$$\underbrace{\frac{\partial T_2}{\partial R_1}\frac{dR_1}{d\delta}}_{\text{Liability-side regulation}} + \underbrace{\sum_j \frac{\partial T_2}{\partial \psi_j}\frac{d\psi_j}{d\delta}}_{\text{Asset-side regulation}}$$

As above, we expect $\frac{\partial T_2}{\partial R_1}\frac{dR_1}{d\delta} > 0$, making more desirable a low level of deposit insurance. However, at this level of generality it is impossible to individually sign the terms corresponding to $\frac{\partial T_2}{\partial \psi_j}\frac{d\psi_j}{d\delta}$. We could expect the sum all these terms to be negative, since banks have an incentive to take more risks in those assets which pay more in no failure states. However, previous research has shown that the risk taking behavior of banks is sensitive to the details of the market environment; see, for instance, Boyd and De Nicolo (2005) and Martinez-Miera and Repullo (2010) on the importance of bank's franchise value. A full analysis of banks risk taking behavior is tangential to our main question and outside the scope of this paper.

In this more general environment, both liability side regulation, controlling the deposit rate offered by banks, and asset side regulations, controlling the investment portfolio of banks are in general needed to maximize social welfare when ex-ante policies are feasible. The optimal corrective policy introduces wedges on banks choices given by

$$\tau_{R_1} \equiv q\mathbb{E}_R\left[\left(1-\lambda\right)U'\left(C_{2R}\right)\frac{\partial T_2}{\partial R_1}\right] \quad \text{and} \quad \tau_{\psi_j} \equiv q\mathbb{E}_R\left[\left(1-\lambda\right)U'\left(C_{2R}\right)\frac{\partial T_2}{\partial \psi_j}\right]$$

As discussed above, restrictions on the set of ex-ante instruments available to the planner deliver intermediate outcomes between the two extremes analyzed here. These formulas provide direct guidance to how to set ex-ante policies to correct the ex-ante behavioral distortions caused by deposit insurance.

Finally, note that it is never optimal for banks to allocate resources to projects with negative net present values, because they act on behalf of depositors. Any distortion along this margin

would also require ex-ante regulation. Therefore, more generally, as long as any other ex-ante welfare relevant friction exists, but it can be addressed through ex-ante corrective regulation, an equation like (19) will characterize δ^*.

4.2 Alternative equilibrium selection mechanisms

In the basic framework, we assume that depositors coordinate following an exogenous sunspot. We now show that varying the information structure and the equilibrium selection procedure does not change the sufficient statistics we identify. We adopt a global game structure in which late depositors observe at date 1 an arbitrarily precise private signal about the return of the investment by banks $\rho_2(s)$ before deciding $D_{1i}(s)$. With that information structure, Goldstein and Pauzner (2005) show, in a model which essentially corresponds to the ours when $\delta = 0$, that there exists a unique equilibrium in threshold strategies in which depositors withdraw their deposits when they receive a sufficiently low signal but leave their deposits in the bank otherwise.

Since our goal in this paper is to show the robustness of our optimal policy characterization and to directly use the set of sufficient statistics that we identify, we take the outcome of a global game as a primitive. In particular, we take as a prediction of the global game that there exists a threshold $s^G(\delta, R_1)$ such that when $s \leq s^G(\delta, R_1)$ there is a bank failure with certainty but when $s > s^G(\delta, R_1)$, no failure occurs. The threshold has the following properties

$$\frac{\partial s^G}{\partial R_1} \geq 0 \quad \text{and} \quad \frac{\partial s^G}{\partial \delta} \leq 0$$

Goldstein and Pauzner (2005) formally show that $\frac{\partial s^G}{\partial R_1} \geq 0$ and we work under the assumption that $\frac{\partial s^G}{\partial \delta} \leq 0$.[19] In fact, any model of behavior which generates a threshold with these properties, not necessarily a global game, makes our result valid.

Therefore, given the behavior of depositors at date 1, the deposit rate offered by banks at date 0 is the outcome of the maximization of

$$J(R_1;\delta) = \lambda U(R_1 D_0 + Y_1) + (1-\lambda) \left[\begin{array}{l} \int_{\underline{s}}^{s^G(\delta,R_1)} U(C_{2R}(s,\delta,R_1))\, dF(s) \\ + \int_{s^G(\delta,R_1)}^{\bar{s}} U(C_{2N}(s,R_1))\, dF(s) \end{array} \right]$$

In this model, the probability of a bank failure happening is given by $F\left(s^G\right)$ and $C_{2R}(s,\delta,R_1)$ denotes the average per capita consumption when a bank failure occurs. Under these assumptions, we show that all our insights regarding the optimal policy formulas go through.

Proposition 5. (Optimal DI with an alternative equilibrium selection) *a) The optimal level of*

[19] Allen et al. (2014) derive this exact comparative static in a model with general government guarantees.

deposit insurance δ^* is characterized by

$$\delta^* = \frac{\varepsilon_\delta^q \left(U\left(C_{2R}\left(s^G\right)\right) - U\left(C_{2N}\left(s^G\right)\right)\right)}{q\mathbb{E}_R\left[U'\left(C_{2R}\left(s\right)\right)\left(\kappa + \left(1-\left(1+\kappa\right)\chi\rho_2\left(s\right)\right)\mathbb{I}_1 + \frac{\partial T_2}{\partial R_1}\frac{dR_1}{d\delta}\right)\right]}, \tag{20}$$

where $\mathbb{E}_R\left[\cdot\right]$ stands for a conditional expectation over bank failure states and, as defined above, q denotes the unconditional probability of a bank failure happening, $\varepsilon_\delta^q = \frac{\partial q(\delta)}{\partial \log(\delta)}$ denotes the change in the likelihood of bank failure induced by a percent change in the level of deposit insurance and $\mathbb{I}_1 = \mathbb{I}\left(\delta \geq \frac{r_1 D_0}{1-\lambda}\right)$.

b) The optimal level of deposit insurance δ^*, when the policymaker has access to ex-ante regulation, is characterized by

$$\delta^* = \frac{\varepsilon_\delta^q \left(U\left(C_{2R}\left(s^G\right)\right) - U\left(C_{2N}\left(s^G\right)\right)\right)}{q\mathbb{E}_R\left[U'\left(C_{2R}\left(s\right)\right)\left(\kappa + \left(1-\left(1+\kappa\right)\chi\rho_2\left(s\right)\right)\mathbb{I}_1\right)\right]}$$

This expression is identical to the one in proposition 4.

The particular information structure assumed and how it affects the equilibrium selection only affects the level of δ^* through the sufficient statistics we identify in this paper. In particular, the change in likelihood of a bank failure ε_δ^q will be directly affected by the assumptions on the informational structure of the economy and, to make positive predictions, it is desirable to have a model in which the probability of a bank failure is fully endogenous, as Allen et al. (2014). However, the main takeaway of this extension and an important result of our paper is that the optimal formulas we characterize are general and independent of the specific details of the information structure of the economy.[20]

4.3 Aggregate spillovers

So far, because bank decisions have not affected aggregate variables, our analysis, as in Diamond and Dybvig (1983), can be defined as a microprudential. When the decisions made by banks affect aggregate variables, for instance asset prices, further exacerbating the possibility of a bank failure, the optimal deposit insurance formula may incorporate a macroprudential correction. These general equilibrium effects arise in models in which the aggregate consequences of decentralized choices directly exacerbate coordination failures. See, for instance, Ramcharan and Rajan (2014) for an empirical analysis or Parlatore (2014) for a theoretical application of similar forces in a model of money market funds.

We now assume that, given a level of aggregate withdrawals $\overline{\Delta}$, banks must liquidate a proportion $\theta\left(\overline{\Delta}\right) > 1$ of their investments. We assume that $\theta\left(\cdot\right)$ is always greater than unity and well-behaved and it captures the possibility of illiquidity in financial markets when unwinding banks' investment opportunities. We model aggregate linkages in the simplest way, but there

[20] Given that we will argue that ε_δ^q is hard to measure directly, using some of the structure imposed by a particular model may be a fruitful approach.

is scope for richer modeling of interbank markets as in, for instance, Freixas, Martin and Skeie (2011).

Hence, the level of resources available to the depositors of a given bank with individual withdrawals $\Delta(s)$, when the level of aggregate withdrawals is $\overline{\Delta}(s)$, is given by

$$\frac{\rho_2\left(D_0-\theta\left(\overline{\Delta}(s)\right)\Delta(s)\right)}{1-\lambda}=\rho_2\left(\theta\left(\overline{\Delta}(s)\right)D_1(s)+\frac{\left(\left(1-\theta\left(\overline{\Delta}(s)\right)\right)R_1-r_1\right)D_0}{1-\lambda}\right) \tag{21}$$

Equation (21) is a generalization of equation (3), when $\theta(\cdot)>1$, capturing that the unit price of liquidating investments is increasing in the aggregate level of withdrawals. Following the same logic used to solve the basic model, we can define thresholds $\hat{s}$ and s^*, which now have $\overline{\Delta}$ as a new argument. Each individual bank chooses R_1 optimally and the equilibrium value of R_1 is determined by the solution to $\frac{\partial J}{\partial R_1}\Big|_{T_2,\overline{\Delta}}=0$, imposing that $\overline{\Delta}=\Delta$.

When the regulator sets δ optimally, he takes into account the effects of banks individual banks choices on the aggregate level of withdrawals $\overline{\Delta}$. We assume that the $\theta(\cdot)-1$ additional units lost by banks is merely a transfer to a set of unmodeled agents that purchase those assets. Under these assumptions, we show that the optimal level of δ^* incorporates a macroprudential correction when ex-ante policy is not available. As it should be clear by now, an equation like (15) characterizes the optimal level of deposit insurance when ex-ante corrective policies are available.

Proposition 6. (Optimal DI with aggregate spillovers) *a) The optimal level of deposit insurance δ^*, without ex-ante regulation, is characterized by*

$$\delta^*=\frac{\varepsilon_\delta^q\left(U\left(C_{2R}(s^*)\right)-U\left(C_{2N}(s^*)\right)\right)+\left(\varsigma_1+\varsigma_2\right)\frac{dR_1}{d\delta}}{q\mathbb{E}_R\left[U'\left(C_{2R}(s)\right)\left(\kappa+\left(1-(1+\kappa)\chi\rho_2(s)\right)\mathbb{I}_1+\frac{\partial T_2}{\partial R_1}\frac{dR_1}{d\delta}\right)\right]}, \tag{22}$$

where ς_1 and ς_2 are defined as $\varsigma_1\equiv(1-\pi)\left[U\left(C_{2R}(\hat{s}_1)\right)-U\left(C_{2N}(\hat{s}_1)\right)\right]\left(\frac{\partial\hat{s}_1}{\partial R_1}-\frac{\partial\hat{s}_0}{\partial R_1}\right)f(\hat{s}_1)$ and $\varsigma_2\equiv\pi\left[U\left(C_{2R}(s_1^*)\right)-U\left(C_{2N}(s_1^*)\right)\right]\left(\frac{\partial s_1^*}{\partial R_1}-\frac{\partial s_0^*}{\partial R_1}\right)f(s_1^*)$, $\mathbb{E}_R[\cdot]$ stands for a conditional expectation over bank failure states and $\varepsilon_\delta^q=\frac{\partial q(\delta)}{\partial\log\delta}$ denotes the change in the likelihood of a bank failure induced by a one percent change in the level of deposit insurance.

b) The optimal level of deposit insurance δ^*, when the policymaker has access to ex-ante regulation, is characterized by

$$\delta^*=\frac{\varepsilon_\delta^q\left(U\left(C_{2R}(s^*)\right)-U\left(C_{2N}(s^*)\right)\right)}{q\mathbb{E}_R\left[U'\left(C_{2R}(s)\right)\left(\kappa+\left(1-(1+\kappa)\chi\rho_2(s)\right)\mathbb{I}_1\right)\right]}$$

This expression is identical to the one in proposition 6.

The only new term in equation (22) is $(\varsigma_1+\varsigma_2)\frac{dR_1}{d\delta}$ in the numerator, which tilts the optimal deposit insurance level to induce banks to internalize the aggregate spillovers they cause in other banks when offering high deposit rates, which make bank failures more likely. However,

an important takeaway of our analysis is that ex-ante regulation is able to target directly the wedges caused by aggregate spillovers. In this case, the ex-ante regulation faced by banks partly addresses both the fiscal externality that emerges from the presence of deposit insurance and the externality induced by the aggregate spillovers caused by competitive deposit rate setting. Similar formulas would apply when banks have general portfolio decisions, as in our analysis earlier in this section.

4.4 Heterogeneous depositors

In the basic framework, we assume that all depositors are ex-ante identical. However, in practice, the level of deposit holdings varies significantly in the cross section of depositors. We now introduce heterogeneity on the level of deposit holdings by assuming that the initial level of deposits D_{0i} is cross-sectionally distributed according to a cdf $G(\cdot)$ and support $\left[0, \overline{D}\right]$. We allow for the possibility that depositors may have different preferences $U_i(\cdot)$. Now we use the index i to denote a given type of depositor and restrict our attention to equilibria in which all type i depositors adopt symmetric strategies. A fraction λ of depositors becomes an early type, regardless of the initial level of deposits.

Hence, depositors' ex-ante utility V_i is given by

$$V_i = \mathbb{E}_s \left[\lambda U_i\left(C_{1i}(s)\right) + (1-\lambda) U_i\left(C_{2i}(s)\right)\right]$$

Banks set R_1 competitively maximizing an average of depositors' expected utilities — this is the rate set by competitive banks under the veil of ignorance regarding the level of deposits holdings. Formally, R_1 is set as to solve

$$\max_{R_1} \int V_i dG(i)$$

As before, we define the amount of deposits withdrawn by type i depositors as

$$\Delta_i(s) \equiv R_1 D_{0i} - D_{1i}(s),$$

and aggregate net withdrawals as

$$\Delta(s) = \int \Delta_i(s)\, di = R_1 \int D_{0i} dG(i) - \int D_{1i} dG(i),$$

The consumption of a given late depositor at date 2 in state s can be expressed as

$$C_{2i}(s) = \begin{cases} \alpha_1(s)\left(\eta\Delta_{1i}(s) + \min\left\{D_{1i}(s), \delta\right\}\right) + (1-\alpha_1(s))\min\left\{R_1 D_{0i}, \delta\right\} + Y_2(s) - T_2(s), & \text{Failure at date 1} \\ \eta\Delta_{1i}(s) + \min\left\{D_{1i}(s), \delta\right\} + \alpha_{2R}(s)\max\left\{D_{1i}(s) - \delta, 0\right\} + Y_2(s) - T_2(s), & \text{Failure at date 2} \\ \eta\Delta_{1i}(s) + \alpha_{2N}(s) D_{1i}(s) + Y_2(s) - T_2(s), & \text{No Bank Failure} \end{cases}$$

Where the scalars $\alpha_1(s) \in [0,1)$, $\alpha_{2R}(s) \in [0,1)$, and $\alpha_{2N}(s) \geq 1$ represent equilibrium objects that depend on the actions of other depositors through the level of available funds $\rho_2(s)\left(\int D_{0i}dG(i) - \Delta(s)\right)$, defining three regions, as follows

$$\text{if } \begin{cases} \rho_2(s)\left(\int D_{0i}dG(i) - \Delta(s)\right) < 0, & \text{Failure at date 1} \\ 0 \leq \rho_2(s)\left(\int D_{0i}dG(i) - \Delta(s)\right) < \int D_{1i}(s)\,dG(i), & \text{Failure at date 2} \\ \rho_2(s)\left(\int D_{0i}dG(i) - \Delta(s)\right) \geq \int D_{1i}(s)\,dG(i), & \text{No Bank Failure} \end{cases}$$

As in the basic framework, it can be shown that there are two different types of candidates for equilibrium. In one (no failure) equilibrium, depositors leave all their funds in the banks, so $D_{1i} = R_1 D_{0i}$. In the other (failure) equilibrium, it is optimal for a given depositor to leave up to the level of deposit insurance, that is, $D_{1i}(s) = \min\{\delta, R_1 D_{0i}\}$. In that case, we can denote the aggregate level of deposits left in the bank by late depositors as D_1^δ

$$D_1^\delta \equiv \int D_{1i}(s)\,dG(i) = (1-\lambda)\left(\int_0^{\frac{\delta}{R_1}} R_1 D_{0i}dG(i) + \delta\int_{\frac{\delta}{R_1}}^{\overline{D}} dG(i)\right) \tag{23}$$

As above, we can express the threshold between the failure and no failure regions as the solution to $\rho_2(s)\left(\int D_{1i}(s)\,dG(i) - r_1\int D_{0i}dG(i)\right) = \int D_{1i}(s)\,dG(i)$, given by

$$D_1^\delta = \frac{1}{1-\frac{1}{\rho_2(s)}}\frac{r_1\int D_{0i}dG(i)}{1-\lambda}, \tag{24}$$

where D_1^δ is defined in equation (23). The solution to equation (24) in δ characterizes $\delta^*(s, R_1)$, which shares the same properties as in the baseline model under natural regularity conditions on $G(\cdot)$. The thresholds $\hat{s}(R_1)$ and $s^*(\delta, R_1)$ are characterized as before, which leaves the expression for $q(\delta, R_1)$ in equation (6) unchanged. The characterization of δ^* is the natural generalization of the baseline case, using cross sectional averages for depositor specific variables.

Proposition 7. (Optimal DI with ex-ante heterogeneous depositors) *a) The optimal level of deposit insurance δ^*, without ex-ante regulation, is characterized by*

$$\delta^* = \frac{\varepsilon_\delta^q \int \left[U\left(C_{2i}^R(s^*)\right) - U\left(C_{2i}^N(s^*)\right)\right] dG(i)}{q\mathbb{E}_R\left[\int U'\left(C_{2i}^R\right)\left(\frac{\partial C_{2i}^R}{\partial \delta} - \frac{\partial T_{2i}}{\partial R_1}\frac{dR_1}{d\delta}\right) dG(i)\right]},$$

where $\mathbb{E}_R[\cdot]$ stands for a conditional expectation over bank failure states and $\varepsilon_\delta^q = \frac{\partial q(\delta)}{\partial \log \delta}$ denotes the change in the likelihood of bank failure induced by a one percent change in the level of deposit insurance.

The introduction of cross-sectional heterogeneity in the level of deposits changes by little the characterization of the equilibrium and it simply extends the optimal DI formula to account for the welfare of heterogeneous depositors.

5 Combining theory and measurement

The approach developed in this paper allows us to link the theoretical tradeoffs that determine the optimal deposit insurance policy to a small number of observables. To show the applicability of our results in practice, we now study the quantitative implications of our results for the optimal coverage level in practice. We use a yearly calibration.

A workable approximation As a benchmark, and partly because it requires a minimum amount of information, we exclusively focus on understanding the implications of our model for the optimal level of deposit insurance when the policymaker has access to ex-ante corrective instruments. To further sharpen our analysis, we use the following approximation.

Proposition 8. (An approximation for δ^*) *The optimal level of deposit insurance, can be approximated by*

$$\delta^* \approx \frac{\varepsilon_\delta^q \left(C_{2N}\left(s^*\right) - C_{2R}\left(s^*\right)\right)}{q\left(1+\kappa\right)\mathbb{E}_R\left[\frac{U'(C_{2R}(s))}{U'(C_{2R}(s^*))}\left(1-\chi\rho_2\left(s\right)\right)\right]}, \tag{25}$$

where q is probability of bank failure, κ is the net marginal cost of public funds, ε_δ^q is the partial semielasticity of the probability of bank failure with respect to the deposit insurance limit and $C_{2R} - C_{2N}$ is the drop in depositors consumption caused by a bank failure, and $\mathbb{E}_R\left[\frac{U'(C_{2R}(s))}{U'(C_{2R}(s^*))}\left(1-\chi\rho_2\left(s\right)\right)\right]$ denotes the appropriately discounted net marginal return of leaving a dollar of deposits inside a bank whenever deposit insurance has to actually be paid.

This simple approximation, which allows us to write δ^* as function of five terms, replaces the marginal benefit in welfare terms of reducing bank failures, given by $U\left(C_{2R}\left(s^*\right)\right) - U\left(C_{2N}\left(s^*\right)\right)$, for its first-order approximation, which allows us to make comparisons in marginal terms. It also assumes that δ is large enough at the optimum so that $\delta > \frac{r_1 D_0}{1-\lambda}$ is verified.

Calibration Three of the variables that appear in equation (25) have clear direct counterparts. They are given in table 1. First, we use average yearly probability of bank failure from 1934 until 2014 using FDIC data, which is 0.436% per year. Second, there are a number of classic estimates of the net marginal cost of public funds, which oscillate between 9% and 14% — see, for instance, Hendren (2013) for a recent discussion of reasonable estimates of κ. We take an average value of 11%. Third, we take as representative the average balance per deposit account from the recent IndyMac failure as a representative bank — see Iyer and Puri (2012). According to the Office of Thrift Supervision, their average balance per deposit account at closure was of approximately of $\$70,000$. Using a recovery rate of 75%, consistent with FDIC data, we use a value of $C_{2R}\left(s^*\right) - C_{2N}\left(s^*\right) \approx -\$17,500$ for the average deposit loss at the marginal state s^*.[21]

[21] For reference, the aggregate level of deposits is 12 trillion, the number of commercial banks (with a steep decreasing trend) is 5,000 and US population older than 18 is approximately 234 million people. A back-of-the-

Variable	Description	Value
q	Probability of bank failure	0.436%
κ	Net marginal cost of public funds	11%
$C_{2R} - C_{2N}$	Consumption drop induced by bank failure	$-17,500$
$\mathbb{E}_R \left[\frac{U'(C_{2R}(s))}{U'(C_{2R}(s^*))} (1 - \chi\rho_2(s)) \right]$	Priced social gain from keeping resources inside banks	1

Table 1: Calibrated variables

Finding a value for the two remaining variables, $\mathbb{E}_R \left[\frac{U'(C_{2R}(s))}{U'(C_{2R}(s^*))} (1 - \chi\rho_2(s)) \right]$ and ε_δ^q, is less straightforward. First, note that $\frac{U'(C_{2R}(s))}{U'(C_{2R}(s^*))}$ corresponds to the relative valuation of a dollar in state s relative to state s^*, which is the marginal state in which bank failures cease to occur. Using a CRRA utility assumption, we can write this term as $\left(\frac{C_{2R}(s)}{C_{2R}(s^*)} \right)^{-\gamma}$, where we know that $\frac{C_{2R}(s)}{C_{2R}(s^*)} < 1$. Unsurprisingly, this term is sensitive to assumptions on the risk aversion coefficient γ. Using the accepted value in finance of $\gamma = 10$ and assuming a consumption drop of 15%, we find that depositors roughly value a 25% more having a dollar in bank failure states relative to the marginal state s^*. The term $1 - \chi\rho_2(s)$ is a function of the average return on assets for banks, of approximately $\rho_2 - 1 \approx 1\%$ (from FRED), and a recovery rate of $\chi = 75\%$, to be consistent with our assumption above. Hence, combining both figures, it is reasonable to assume that the term $\mathbb{E}_R \left[\frac{U'(C_{2R}(s))}{U'(C_{2R}(s^*))} (1 - \chi\rho_2(s)) \right]$ takes approximately a unit value.

Finally, the main challenge of this calibration is to find appropriate values for ε_δ^q, the sensitivity of the likelihood of having to actually use the deposit insurance guarantee with respect to the coverage level. Conceptually, there are two ways of recovering this value. First, a direct approach is to measure ε_δ^q directly. This can be done by running a regression of the type:[22]

$$Y = \beta_0 \log(\delta) + \beta_1 X + u,$$

where Y is an indicator for actual FDIC assistance events and X are control variables. The estimate of the marginal effect of δ, which exactly corresponds to β_0 in this case, recovers ε_δ^q. A second alternative approach is to run a similar regression using inferred values for the likelihoods of bank failure as dependent variable, proxied by credit risk premia or CDS values. It is challenging for both approaches to overcome endogeneity problems.

Recovering implied elasticities An alternative way of exploiting our results is to use equation (25) to recover the implied bank failure sensitivity ε_δ^q using the observed values of deposit insurance coverage. This approach assumes that deposit insurance limits are set following the logic of our model. We carry out two simple policy experiments trying to rationalize the change in deposit insurance coverage observed in 2008.

envelope calculation finds that the representative depositor should have \$51,000 dollars in deposits.

[22] We are using a linear probability model to ease the illustration of the results. More sophisticated binary dependent variable models (e.g., Probit or Logit) can be more appropriate in practice.

First, using the long run averages of table 1, we use equation (25) to recover the implied values of ε_{δ}^{q} before and after the last policy change, that is $\delta = 100,000$ and $\delta = 250,000$. This exercise yields

$$\begin{aligned} &\text{if} \quad \delta = 100,000, \qquad \varepsilon_{\delta}^{q} \approx 0.0277 \\ &\text{if} \quad \delta = 250,000, \qquad \varepsilon_{\delta}^{q} \approx 0.0691 \end{aligned}$$

Therefore, a 10% increase in δ (equal to $10,000$ or $25,000$ respectively) is associated with a reduction in the likelihood of failure q of 0.277% or 0.691% respectively. The values that we recover are reasonable, given that the variable q has a mean of 0.4% with a maximum of 4.2%.

A second exercise can allow us to rationalize the policy change observed in 2008. Assuming that q was at its historical mean before 2008, that there was a sudden exogenous change to the probability of bank failure at 2008 and that after the change in policy, the likelihood of bank failure remains at its historical mean, we can solve for the implied magnitude of the exogenous jump in probability, which we denote by θ, by solving[23]

$$0.00436 = \theta 0.00436 - 0.0484 \times 1.5 \Rightarrow \theta \approx 15$$

This value of θ implies that the expected likelihood of bank failure without intervention would have increased fifteenfold up to 6.54%, a number that is approximately 50% higher than the historical maximum. Several other exercises could help to rationalize the change in the level of coverage, for instance, changes in the losses caused by a bank failure or changes in the fiscal outlook. Looking forward, only precise measurement of the variables we identify in this paper can provide a definite answer regarding the optimality of policy decisions.

Before concluding, we would like to qualify the validity of the quantitative conclusions of this section. Even though our theoretical results, as in propositions 1 and 2 and in the extensions, are exact and do not rely on approximations, when using them quantitatively we incur in two types of approximation errors. First, when using the approximation developed in proposition 7, we might be distorting the curvature of preferences; this should not change much the results. Second, and more importantly, when holding constant the variables used to calibrate δ^*, we implicitly assume that they do not vary when δ changes; this can be a more important source of error, mainly when we discuss large changes in δ. If we could recover flexible estimates of all variables for all levels of δ, this would not be a concern but, as of today, that is unfeasible. Otherwise, any practical use of an equation like (25) must take into consideration these errors.

[23] We use the average value of the elasticities before and after the policy change, that is, $\varepsilon_{\delta}^{q} = \frac{0.0691+0.0277}{2} = 0.0484$.

6 Conclusion

We have developed a theoretical characterization of the optimal level of deposit insurance that applies to a wide variety of environments. Our model allows us to identify the set of variables which have a first-order effect on welfare and that become sufficient statistics for assessing policy changes. In that regard, we provide a step forward towards building a microfounded *theory of measurement* for financial regulation.

Building on our framework, there are a number of avenues for further research. From a theoretical perspective, allowing for a rich cross-section of depositors with different characteristics or exploring alternative forms of competition among heterogeneous banks are natural non-trivial extensions. Although perhaps the most promising implications of this paper for future research come from the measurement perspective. Recovering robust, well-identified and credible estimates in different contexts of the sufficient statistics we have identified in this paper, in particular of ε_{δ}^{q}, has the potential to directly discipline future regulatory choices.

Appendix

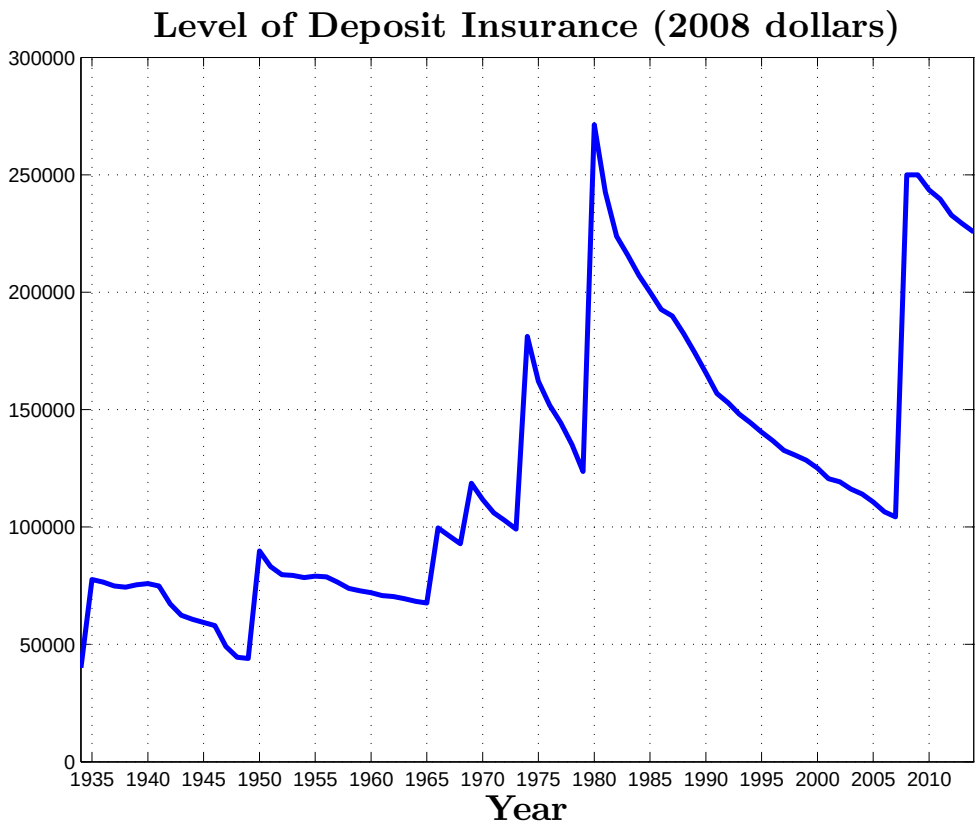

Figure 6: Level of Deposit Insurance (1934-2014, measured in dollars of 2008)

Figure 6 plots the evolution level of deposit insurance measured in dollars of 2008 using a CPI deflator.

Proofs: Section 2

Under the sustained convention that $D_1(s)$ denotes the deposit level of late depositors, we can write

$$\begin{aligned} D_0 - \Delta(s) &= D_0 - R_1 D_0 + (1-\lambda) D_1(s) \\ &= -r_1 D_0 + (1-\lambda) D_1(s), \end{aligned}$$

which allows us to derive equation (3) in the text. We represent equation (3) in figure 7. Whenever $\rho_2(s) > 1$, there exists a value of $D_1(s)$ that defines a threshold between the failure and no failure regions. It is given by the solution to $\frac{\rho_2(s)(D_0 - \Delta(s))}{1-\lambda} = D_1(s)$. When $D_1(s) = \delta$, the threshold characterizes $\delta^*(s, R_1)$.

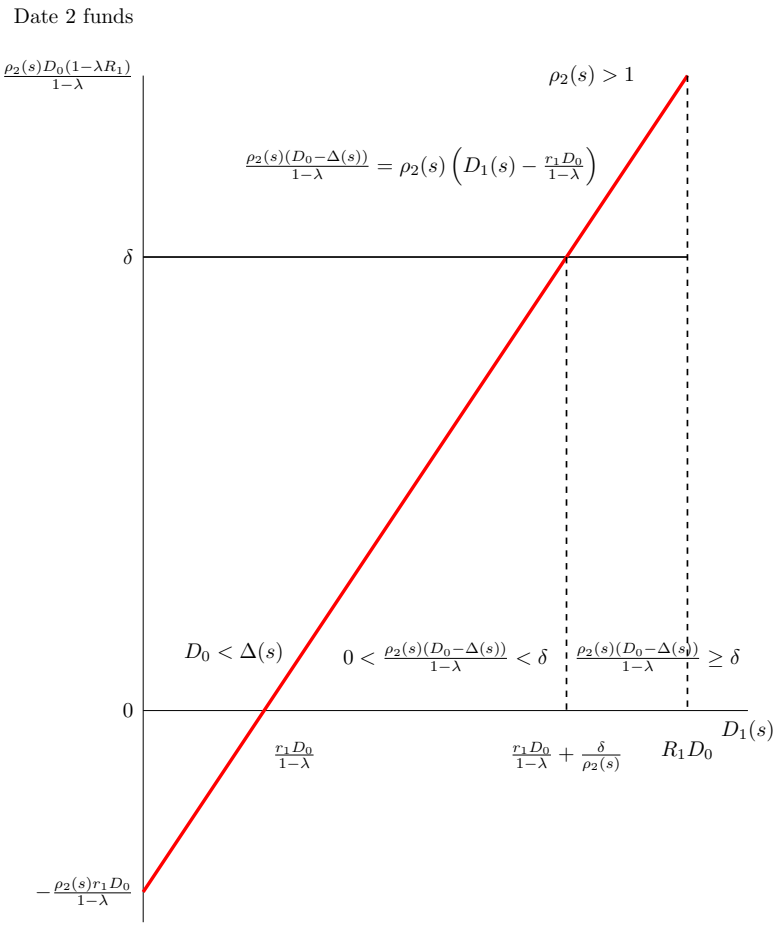

Figure 7: Date 2 funds for a given $D_1(s)$ choice of late depositors in a symmetric equilibrium

Properties of $\delta^*(s, R_1)$

We can define $\phi(s) \equiv \frac{1}{1-\frac{1}{\rho_2(s)}} \frac{1}{1-\lambda}$, with $\frac{\partial \phi(s)}{\partial s} < 0$, given our assumptions on $\rho_2(s)$ and λ. Hence

$$\delta^*(s, R_1) = \phi(s) r_1 D_0$$

So

$$\frac{\partial \delta^*}{\partial s} = \frac{\partial \phi(s)}{\partial s} r_1 D_0 < 0 \qquad \text{and} \qquad \frac{\partial \delta^*}{\partial R_1} = \phi(s) D_0 > 0$$

Properties of $\hat{s}(R_1)$, $s^*(\delta, R_1)$ and $q(\delta, R_1)$

We can write $\min\{\delta, D_0 R_1\} = \phi(s^*) r_1 D_0$ and $R_1 = \phi(\hat{s}) r_1$. Hence

$$\frac{\partial s^*}{\partial \delta} = \begin{cases} \frac{1}{\frac{\partial \phi}{\partial s} r_1 D_0} < 0, & \text{if } \delta < D_0 R_1 \\ 0, & \text{if } \delta \geq D_0 R_1 \end{cases}$$

$$\frac{\partial s^*}{\partial R_1} = \begin{cases} \frac{-\phi(s)}{\frac{\partial \phi}{\partial s} r_1} > 0, & \text{if } \delta < D_0 R_1 \\ \frac{\partial \hat{s}}{\partial R_1} > 0, & \text{if } \delta \geq D_0 R_1 \end{cases}$$

$$\frac{\partial \hat{s}}{\partial R_1} = \frac{1 - \phi(s)}{\frac{\partial \phi}{\partial s} r_1} > 0$$

Note that $\frac{ds^*(\delta, R_1)}{d\delta} = \frac{\partial s^*}{\partial \delta} + \frac{\partial s^*}{\partial R_1} \frac{dR_1}{d\delta}$.

We can write equation (6) as

$$q(\delta, R_1) = F(\hat{s}(R_1)) + \pi [F(s^*(\delta, R_1)) - F(\hat{s}(R_1))]$$

We can then show that

$$\frac{\partial q}{\partial \delta} = \pi f(s^*(\delta, R_1)) \frac{\partial s^*}{\partial \delta} \leq 0$$
$$\frac{\partial q}{\partial R_1} = (1 - \pi) f(\hat{s}(R_1)) \frac{\partial \hat{s}}{\partial R_1} + \pi f(s^*(\delta, R_1)) \frac{\partial s^*}{\partial R_1} > 0,$$

using the results derived above that $\frac{\partial s^*}{\partial R_1} > 0$ and $\frac{\partial \hat{s}}{\partial R_1} > 0$.

Properties of $C_{2R}(s, \delta, R_1)$, $\tilde{C}_{2R}(s, \delta, R_1)$ and $T_2(s, \delta, R_1)$

In a failure equilibrium, in which $D_{1i}(s) = \delta$, we can write consumption before taxes as

$$\tilde{C}_{2R}(s, \delta, R_1) = \alpha_1(s, \delta, R_1) R_1 D_0 + (1 - \alpha_1(s, \delta, R_1)) \delta + Y_2(s) = \delta + \alpha_1(s, \delta, R_1)(D_0 R_1 - \delta) + Y_2(s)$$

where $\alpha_1(s, \delta, R_1) = \min\left\{\frac{D_0(1-\lambda R_1)}{(1-\lambda)(R_1 D_0 - \delta)}, 1\right\}$ is the probability of being able to withdraw funds given δ. Hence, we can write

$$\tilde{C}_{2R}(s, \delta, R_1) = \left[\delta + \frac{D_0(1 - \lambda R_1)}{1 - \lambda}\right](1 - \mathbb{I}_1) + D_0 R_1 \mathbb{I}_1 + Y_2(s),$$

where $\mathbb{I}_1 = \mathbb{I}\left[\delta \geq \frac{r_1 D_0}{1-\lambda}\right]$. The amount of fiscal revenue needed is given by

$$T_2(s, \delta, R_1) = (1 + \kappa)\left[\min\{\delta, D_0 R_1\} - \chi \rho_2(s)\left(\min\{\delta, D_0 R_1\} - \frac{r_1 D_0}{1 - \lambda}\right)\mathbb{I}_1\right]$$

It then follows, in the relevant interior case with $\delta < D_0 R_1$, after some algebra, that

$$\begin{aligned} C_{2R}(s, \delta, R_1) &= \left[\delta + \frac{D_0(1 - \lambda R_1)}{1 - \lambda}\right](1 - \mathbb{I}_1) + \left[D_0 R_1 + (1 + \kappa)\chi \rho_2(s)\left(\delta - \frac{r_1 D_0}{1 - \lambda}\right)\right]\mathbb{I}_1 + Y_2(s) - (1 + \kappa)\delta \\ &= \frac{D_0(1 - \lambda R_1)}{1 - \lambda} + ((1 + \kappa)\chi \rho_2(s) - 1)\left(\delta - \frac{r_1 D_0}{1 - \lambda}\right)\mathbb{I}_1 + Y_2(s) - \kappa \delta \end{aligned}$$

We can thus derive the following comparative statics, which are important inputs for the equilibrium characterizations

$$\frac{\partial \tilde{C}_{2R}}{\partial R_1} = \left[-\frac{\lambda}{1 - \lambda}(1 - \mathbb{I}_1) + \mathbb{I}_1\right] D_0$$

$$\frac{dC_{2N}}{dR_1} = -\rho_2(s)\frac{\lambda}{1-\lambda}D_0 < 0$$

$$\frac{\partial T_2}{\partial \delta} = (1+\kappa)\left[1-\chi\rho_2(s)\mathbb{I}_1\right]\mathbb{I}\left[\delta < D_0R_1\right] \gtreqless 0$$
$$\frac{\partial T_2}{\partial R_1} = (1+\kappa)\frac{\chi\rho_2(s)D_0}{1-\lambda}\mathbb{I}_1 \geq 0$$

$$\frac{dC_{2R}}{dR_1} = \left[-\frac{\lambda}{1-\lambda} - \frac{(1+\kappa)\chi\rho_2(s)-1}{1-\lambda}\mathbb{I}_1\right]D_0$$
$$\frac{\partial C_{2R}}{\partial \delta} = -\kappa - (1-(1+\kappa)\chi\rho_2(s))\mathbb{I}_1$$

Finally, note that we can write $C_{2N}(s) - C_{2R}(s)$ as

$$C_{2N}(s) - C_{2R}(s) = (\rho_2(s)-1)\frac{D_0(1-\lambda R_1)}{1-\lambda} - ((1+\kappa)\chi\rho_2(s)-1)\left(\min\{\delta, D_0R_1\} - \frac{r_1D_0}{1-\lambda}\right)\mathbb{I}_1 + \kappa\delta$$

$$C_{2N}(s) - C_{2R}(s) = (\rho_2(s)-1)\left[\frac{D_0(1-\lambda R_1)}{1-\lambda} - \left(\delta - \frac{r_1D_0}{1-\lambda}\right)\mathbb{I}_1\right] - ((1+\kappa)\chi-1)\rho_2(s)\left(\delta - \frac{r_1D_0}{1-\lambda}\right)\mathbb{I}_1 + \kappa\delta$$

The first terms in the last equation is necessarily positive, since $\delta \leq D_0R_1$. Hence, a sufficient condition for $C_{2N}(s) - C_{2R}(s) > 0, \ \forall s$ is that $\chi < \frac{1}{1+\kappa}$, which necessarily holds, given our assumptions.

Choice of $R_1^*(\delta)$ by banks

Zero profit competitive banks choose R_1 to maximize, for a given level of deposit insurance δ, equation (9) stated in the text, taking T_2 as given. The first order condition for banks is $\frac{\partial J}{\partial R_1}\Big|_{T_2} = 0$, given in equation (10) in the text, can be written as

$$\begin{aligned}
\frac{\partial J}{\partial R_1}\Big|_{T_2} &= \lambda U'(R_1D_0)D_0 + (1-\lambda)\int_{\underline{s}}^{\hat{s}} U'(C_{2R}(s))\left[-\frac{\lambda}{1-\lambda}(1-\mathbb{I}_1)+\mathbb{I}_1\right]D_0dF(s) \\
&+ (1-\lambda)\int_{\hat{s}}^{s^*}\left(\begin{array}{c}\pi U'(C_{2R}(s))\left[-\frac{\lambda}{1-\lambda}(1-\mathbb{I}_1)+\mathbb{I}_1\right]D_0 \\ +(1-\pi)U'(C_{2N}(s))\left[-\frac{\lambda}{1-\lambda} - \frac{(1+\kappa)\chi\rho_2(s)-1}{1-\lambda}\mathbb{I}_1\right]D_0\end{array}\right)dF(s) \\
&+ (1-\lambda)\int_{s^*}^{\bar{s}} U'(C_{2N}(s))\left[-\frac{\lambda}{1-\lambda} - \frac{(1+\kappa)\chi\rho_2(s)-1}{1-\lambda}\mathbb{I}_1\right]D_0dF(s) \\
&+ (1-\lambda)(1-\pi)\left[U(C_{2R}(\hat{s})) - U(C_{2N}(\hat{s}))\right]\frac{\partial \hat{s}}{\partial R_1}f(\hat{s}) \\
&+ (1-\lambda)\pi\left[U(C_{2R}(s^*)) - U(C_{2N}(s^*))\right]\frac{\partial s^*}{\partial R_1}f(s^*)
\end{aligned} \tag{26}$$

Intuitively, when the level of deposit D_0 increases, the marginal terms are more important those corresponding to the change of regime. It is clear that $J(\cdot)$ is differentiable almost everywhere, so equation (26).

For the solution to $\frac{\partial J}{\partial R_1}\Big|_{T_2} = 0$ to be an optimum, it must be that $\frac{\partial J^2(\delta,R_1)}{\partial R_1^2} < 0$ at that point. As we can see in the numerical example in the appendix, the problem solved by banks is not

globally convex. In general, there are no simple sufficient conditions for convexity. In practice, the problem solved by banks is well-behaved for standard choices of utility and distributions. The characterization of $\frac{dR_1}{d\delta}$ is given by

$$\frac{dR_1}{d\delta} = \frac{\frac{\partial^2 J(\delta,R_1)}{\partial R_1 \partial \delta}}{-\frac{\partial J^2(\delta,R_1)}{\partial R_1^2}}$$

At an interior optimum, the denominator is necessarily negative, so the sign of $\frac{dR_1}{d\delta}$ depends on $\frac{\partial^2 J(\delta,R_1)}{\partial R_1 \partial \delta}$, whose detailed derivation is available under request. From equation (26), it is easy to see that there are no substitution effects associated with δ, only income effects, which operate through C_{2R}, and direct effects, which operate through s^* and $\frac{\partial s^*}{\partial R_1}$.

Proofs: Section 3

Proposition 1. (Marginal effect of varying δ on welfare)

We can write social welfare $W(\delta)$ in the following way

$$W(\delta) = \lambda U\left(R_1^*(\delta) D_0 + Y_1\right) + \\ + (1-\lambda)\begin{bmatrix} \int_{\underline{s}}^{\hat{s}\left(R_1^*(\delta)\right)} U\left(C_{2R}\left(s,\delta,R_1^*(\delta)\right)\right) dF(s) \\ + \int_{\hat{s}\left(R_1^*(\delta)\right)}^{s^*\left(\delta,R_1^*(\delta)\right)} \left(\pi U\left(C_{2R}\left(s,\delta,R_1^*(\delta)\right)\right) + (1-\pi) U\left(C_{2N}\left(s,R_1^*(\delta)\right)\right)\right) dF(s) \\ + \int_{s^*\left(\delta,R_1^*(\delta)\right)}^{\overline{s}} U\left(C_{2N}\left(s,R_1^*(\delta)\right)\right) dF(s) \end{bmatrix}$$

Using the results just derived, we can write:

$$\frac{dW}{d\delta} = \lambda U'\left(R_1(\delta) D_0 + Y_1\right) D_0 \frac{dR_1}{d\delta} + (1-\lambda)\int_{\underline{s}}^{\hat{s}} U'\left(C_{2R}\right)\frac{dC_{2R}}{d\delta} dF(s) \\ + (1-\lambda)\int_{\hat{s}}^{s^*}\left(\pi U'\left(C_{2R}\right)\frac{dC_{2R}}{d\delta} + (1-\pi) U'\left(C_{2N}\right)\frac{dC_{2N}}{dR_1}\frac{dR_1}{d\delta}\right) dF(s) \\ + (1-\lambda)\int_{s^*}^{\overline{s}} U'\left(C_{2N}\right)\frac{dC_{2N}}{dR_1}\frac{dR_1}{d\delta} dF(s) \\ + (1-\lambda)\begin{bmatrix} (1-\pi)\left[U\left(C_{2R}\left(\hat{s}\left(R_1\right)\right)\right) - U\left(C_{2N}\left(\hat{s}\left(R_1\right)\right)\right)\right]\frac{\partial \hat{s}(R_1)}{\partial R_1}\frac{dR_1}{d\delta} f\left(\hat{s}\right) \\ +\pi\left[U\left(C_{2R}\left(s^*\left(\delta,R_1\right)\right)\right) - U\left(C_{2N}\left(s^*\left(\delta,R_1\right)\right)\right)\right]\left[\frac{\partial s^*(\delta,R_1)}{\partial R_1}\frac{dR_1}{d\delta} + \frac{\partial s^*(\delta,R_1)}{\partial \delta}\right] f\left(s^*\right) \end{bmatrix}$$

Using the fact that

$$\frac{dC_{2R}}{d\delta} = \frac{\partial C_{2R}}{\partial \delta} + \frac{\partial C_{2R}}{\partial R_1}\frac{dR_1}{d\delta} = \frac{\partial C_{2R}}{\partial \delta} + \left(\frac{\partial \tilde{C}_{2R}}{\partial R_1} - \frac{\partial T_2}{\partial R_1}\right)\frac{dR_1}{d\delta} = \underbrace{\frac{\partial \tilde{C}_{2R}}{\partial R_1}\frac{dR_1}{d\delta}}_{\text{Internalized}} + \underbrace{\frac{\partial C_{2R}}{\partial \delta} - \frac{\partial T_2}{\partial R_1}\frac{dR_1}{d\delta}}_{\text{Not internalized}}$$

combined with the optimality condition on the deposit rate offered by banks, we can write

$$\frac{\frac{dW}{d\delta}}{1-\lambda} = \begin{array}{c} \int_{\underline{s}}^{\hat{s}} U'\left(C_{2R}\right)\left(\frac{\partial C_{2R}}{\partial \delta} - \frac{\partial T_2}{\partial R_1}\frac{dR_1}{d\delta}\right) dF(s) + \pi\int_{\hat{s}}^{s^*} U'\left(C_{2R}\right)\left(\frac{\partial C_{2R}}{\partial \delta} - \frac{\partial T_2}{\partial R_1}\frac{dR_1}{d\delta}\right) dF(s) \\ + \left[U\left(C_{2R}\left(s^*\left(\delta,R_1\right)\right)\right) - U\left(C_{2N}\left(s^*\left(\delta,R_1\right)\right)\right)\right]\pi\frac{\partial s^*(\delta,R_1)}{\partial \delta} f\left(s^*\right) \end{array}$$

To derive equation (12) in the text, we multiply and divide the first element of $\frac{\frac{dW}{d\delta}}{1-\lambda}$ by the probability of bank failure, defined in equation (6), to be able to define a condition expectation, so we define $\mathbb{E}_R[\cdot]$ as

$$\mathbb{E}_R\left[U'(C_{2R})\left(\frac{\partial C_{2R}}{\partial\delta}-\frac{\partial T_2}{\partial R_1}\frac{dR_1}{d\delta}\right)\right]\equiv\frac{\int_{\underline{s}}^{\hat{s}}U'(C_{2R})\left(\frac{\partial C_{2R}}{\partial\delta}-\frac{\partial T_2}{\partial R_1}\frac{dR_1}{d\delta}\right)dF(s)+\int_{\hat{s}}^{s^*}U'(C_{2R})\left(\frac{\partial C_{2R}}{\partial\delta}-\frac{\partial T_2}{\partial R_1}\frac{dR_1}{d\delta}\right)\pi dF(s)}{q(\delta,R_1)}$$

Using the fact that

$$\frac{\partial q(\delta)}{\partial\delta}=\pi f(s^*(\delta,R_1))\frac{\partial s^*(\delta,R_1)}{\partial\delta}$$

As usual in normative exercises — see Atkinson and Stiglitz (1980) or Ljungqvist and Sargent (2004) for detailed discussions in a number of different contexts, it is hard to guarantee the convexity of the planning problem in general: there are no simple conditions on primitives that guarantee the convexity of the planning problem. In practice, for natural parametrizations, as the one presented in our in numerical section, $W(\delta)$ is well-behaved with an interior optimum, after we account for the specific non-convexity discussed in our remarks.

For our remark, we need to argue that $\lim_{\delta\to0^+}\frac{dW}{d\delta}<0$. Using the fact that $\lim_{\delta\to0^+}\frac{\partial s^*(\delta,R_1)}{\partial\delta}=0$ and that $\lim_{\delta\to0^+}\mathbb{I}_1=0$, we can write

$$\lim_{\delta\to0^+}\frac{dW}{d\delta}=-q\mathbb{E}_R\left[(1-\lambda)U'(C_{2R}(s))\right]\kappa<0$$

Hence, as long as there is a fiscal cost of paying for deposit insurance, that is, $\kappa>0$, social welfare is decreasing when $\delta\to0$.

Proposition 2. (Optimal deposit insurance δ^*)

At an interior optimum, the optimal level of deposit insurance is given by $\frac{dW}{d\delta}=0$. Using equation (12), we can write:

$$(U(C_{2R}(s^*))-U(C_{2N}(s^*)))\underbrace{\frac{\partial q(\delta)}{\partial\delta}\delta}_{\equiv\varepsilon_\delta^q}\frac{1}{\delta}=q\mathbb{E}_R\left[U'(C_{2R}(s))\left(\kappa+(1-(1+\kappa)\chi\rho_2(s))\mathbb{I}_1+\frac{\partial T_2}{\partial R_1}\frac{dR_1}{d\delta}\right)\right]$$

And we can solve for δ^* as

$$\delta^*=\frac{\varepsilon_\delta^q(U(C_{2R}(s^*))-U(C_{2N}(s^*)))}{q\mathbb{E}_R\left[U'(C_{2R}(s))\left(\kappa+(1-(1+\kappa)\chi\rho_2(s))\mathbb{I}_1+\frac{\partial T_2}{\partial R_1}\frac{dR_1}{d\delta}\right)\right]},$$

which corresponds to equation (14) in the text.

Proposition 3. (Optimal deposit insurance with ex-ante corrective policies)

The planner's optimality condition are given by $\frac{\partial J}{\partial R_1} = 0$ and $\frac{\partial J}{\partial \delta} = 0$. Formally:

$$\begin{aligned}\frac{\partial J}{\partial R_1} &= \lambda U'(R_1 D_0) D_0 + (1-\lambda)\int_{\underline{s}}^{\hat{s}} U'(C_{2R}(s))\frac{\partial C_{2R}(s)}{\partial R_1} dF(s) \\ &+ (1-\lambda)\left[\int_{\hat{s}}^{s^*}\left(\begin{array}{c}\pi U'(C_{2R}(s))\frac{\partial C_{2R}(s)}{\partial R_1} \\ +(1-\pi)U'(C_{2N}(s))\frac{dC_{2N}(s)}{dR_1}\end{array}\right)dF(s) + \int_{s^*}^{\bar{s}} U'(C_{2N}(s))\frac{dC_{2N}(s)}{dR_1}dF(s)\right] \\ &+ (1-\lambda)(1-\pi)\left[U(C_{2R}(\hat{s})) - U(C_{2N}(\hat{s}))\right]\frac{\partial \hat{s}}{\partial R_1} f(\hat{s}) \\ &+ (1-\lambda)\pi\left[U(C_{2R}(s^*)) - U(C_{2N}(s^*))\right]\frac{\partial s^*}{\partial R_1} f(s^*),\end{aligned} \tag{27}$$

and

$$\frac{\frac{\partial W}{\partial \delta}}{1-\lambda} = \begin{array}{c}\int_{\underline{s}}^{\hat{s}} U'(C_{2R})\frac{\partial C_{2R}}{\partial \delta}dF(s) + \pi\int_{\hat{s}}^{s^*} U'(C_{2R})\frac{\partial C_{2R}}{\partial \delta}dF(s) \\ +\left[U(C_{2R}(s^*(\delta,R_1))) - U(C_{2N}(s^*(\delta,R_1)))\right]\pi\frac{\partial s^*(\delta,R_1)}{\partial \delta} f(s^*)\end{array}$$

Following identical steps to those proving proposition 2, we find equation (15). By comparing equation (26) with (27) and taking the difference, we find the wedge τ_{R_1} in equation (16) in the text

Proofs: Section 4

Proposition 4. (Optimal DI with general portfolio and investment decisions)

The resources at date 2 for a bank are given by $\sum_j \rho_{2j}(s)\left(\psi_j D_0 - \varphi\psi_j D_0\right)$. But the fraction of assets to liquidate is given by the level of withdrawals $\Delta(s)$. Hence, $\Delta(s) = \varphi\sum\psi_j D_0 = \varphi D_0$. Hence, we can write

$$\begin{aligned}\sum_j \rho_{2j}(s)\left(\psi_j D_0 - \varphi\psi_j D_0\right) &= \sum_j \rho_{2j}(s)\psi_j\left(D_0 - \Delta(s)\right) \\ &= \sum_j \rho_{2j}(s)\psi_j\left(D_1 - \frac{r_1 D_0}{1-\lambda}\right)\end{aligned}$$

Where we use the fact that $\varphi = \frac{\Delta(s)}{\sum\psi_j D_0} = \frac{\Delta(s)}{D_0}$. The indifference point in this case is given by $\delta = \sum_j \rho_{2j}(s)\psi_j\left(\delta - \frac{r_1 D_0}{1-\lambda}\right)$, so we can write δ^* as a function of R_1 and ψ_j as

$$\delta^* = \frac{1}{1 - \frac{1}{\sum_j \rho_{2j}(s)\psi_j}}\frac{r_1 D_0}{1-\lambda}$$

We derive our results in the most general form. We define consumption for early and late depositors as

$$C_1(s, R_1, y_k) = R_1 D_0 + \sum_k \rho_{1k}(s) y_k$$

$$C_{2N}\left(s,\delta,R_1,y_k,\psi_j\right)=\frac{\sum_j \rho_{2j}\left(s\right)\psi_j D_0\left(1-\lambda R_1\right)}{1-\lambda}+\sum_k \rho_{2k}\left(s\right)y_k$$

$$C_{2R}\left(s,\delta,R_1,y_k,\psi_j\right)=\tilde{C}_{2R}\left(s,\delta,R_1,D_0,y_k\right)-T_2\left(s,\delta,R_1,D_0,\left\{\psi_j\right\}\right),$$

with $\tilde{C}_{2R}$ given by

$$\tilde{C}_{2R}\left(s,\delta,R_1,y_k\right)=\left[\delta+\frac{D_0\left(1-\lambda R_1\right)}{1-\lambda}\right]\left(1-\mathbb{I}_1\right)+D_0R_1\mathbb{I}_1+\sum_k \rho_{2k}\left(s\right)y_k,$$

and $T_2\left(s,\delta,R_1,\psi_j\right)$ defined in the text.

Hence, at date 0 depositors ex-ante welfare, which determines the choices of R_1, ψ_j and y_k is given by

$$J\left(R_1,y_k,\psi_j;\delta\right)=U\left(Y_0-D_0-\sum_k y_k\right)+\lambda\int_{\underline{s}}^{\bar{s}}U\left(C_1\left(s\right)\right)dF\left(s\right)+\left(1-\lambda\right)\int_{\underline{s}}^{\hat{s}}U\left(C_{2R}\left(s\right)\right)dF\left(s\right)$$
$$+\left(1-\lambda\right)\left[\int_{\hat{s}}^{s^*}\left(\pi U\left(C_{2R}\left(s\right)\right)+\left(1-\pi\right)U\left(C_{2N}\left(s\right)\right)\right)dF\left(s\right)+\int_{s^*}^{\bar{s}}U\left(C_{2N}\left(s\right)\right)dF\left(s\right)\right]$$

Although we do not make it explicit, note that $\hat{s}\left(R_1,\psi_j\right)$ now depends on D_0 and ψ_j. The same occurs with $s^*\left(\delta,R_1,\psi_j\right)$. The optimality condition that determines the choice of R_1 is identical to the one in our basic framework. We can thus characterize the new optimal date 0 choices by

$$\left.\frac{\partial J}{\partial y_k}\right|_{T_2}=0,\ \forall k,\quad \left.\frac{\partial J}{\partial \psi_j}\right|_{T_2}=\nu,\ \forall j,\ \text{and}\ \sum_j \psi_j=1$$

where

$$\left.\frac{\partial J}{\partial y_k}\right|_{T_2}=-U'\left(C_0\right)+\lambda\int_{\underline{s}}^{\bar{s}}U'\left(C_1\left(s\right)\right)\rho_{1k}dF\left(s\right)+\left(1-\lambda\right)\int_{\underline{s}}^{\hat{s}}U'\left(C_{2R}\left(s\right)\right)\rho_{2k}dF\left(s\right)$$
$$+\left(1-\lambda\right)\int_{\hat{s}}^{s^*}\left(\pi U'\left(C_{2R}\left(s\right)\right)\rho_{2k}+\left(1-\pi\right)U'\left(C_{2N}\left(s\right)\right)\rho_{2k}\right)dF\left(s\right)$$
$$+\left(1-\lambda\right)\int_{s^*}^{\bar{s}}U'\left(C_{2N}\left(s\right)\right)\rho_{2k}dF\left(s\right)$$

and

$$\left.\frac{\partial J}{\partial \psi_j}\right|_{T_2}=D_0\left(1-\lambda R_1\right)\left[\left(1-\pi\right)\int_{\hat{s}}^{s^*}U'\left(C_{2N}\left(s\right)\right)\rho_{2j}\left(s\right)dF\left(s\right)+\int_{s^*}^{\bar{s}}U'\left(C_{2N}\left(s\right)\right)\rho_{2j}\left(s\right)dF\left(s\right)\right]$$
$$+\left(1-\pi\right)\left[U\left(C_{2R}\left(\hat{s}\right)\right)-U\left(C_{2N}\left(\hat{s}\right)\right)\right]\frac{\partial\hat{s}\left(R_1\right)}{\partial\psi_j}f\left(\hat{s}\right)$$
$$+\pi\left[U\left(C_{2R}\left(s^*\right)\right)-U\left(C_{2N}\left(s^*\right)\right)\right]\frac{\partial s^*\left(\delta,R_1\right)}{\partial\psi_j}f\left(s^*\right)$$

The first K conditions are standard consumption Euler equations for the different assets chosen by depositors. The next $J+1$ conditions are standard optimal portfolio conditions. Note that banks do not internalize the effect of their decisions on the fiscal cost T_2.

Social welfare can thus be written as

$$W(\delta) = U\left(Y_0 - D_0 - \sum_k y_k\right) + \lambda \int_{\underline{s}}^{\bar{s}} U(C_1(s))\, dF(s) + (1-\lambda)\int_{\underline{s}}^{\hat{s}} U(C_{2R}(s))\, dF(s)$$
$$+ (1-\lambda)\left[\int_{\hat{s}}^{s^*} (\pi U(C_{2R}(s)) + (1-\pi)U(C_{2N}(s)))\, dF(s) + \int_{s^*}^{\bar{s}} U(C_{2N}(s))\, dF(s)\right],$$

where R_1, y_k and ψ_j are chosen optimally as a function of δ. We use the fact that $\frac{dC_{2R}}{d\delta}$ can be written as

$$\frac{dC_{2R}}{d\delta} = \frac{\partial C_{2R}}{\partial \delta} + \frac{\partial C_{2R}}{\partial R_1}\frac{dR_1}{d\delta} + \sum_j \frac{\partial C_{2R}}{\partial \psi_j}\frac{d\psi_j}{d\delta} + \sum_k \frac{\partial C_{2R}}{\partial y_k}\frac{dy_k}{d\delta}$$
$$= \underbrace{\frac{\partial \tilde{C}_{2R}}{\partial R_1}\frac{dR_1}{d\delta} + \sum_j \frac{\partial \tilde{C}_{2R}}{\partial \psi_j}\frac{d\psi_j}{d\delta} + \sum_k \frac{\partial \tilde{C}_{2R}}{\partial y_k}\frac{dy_k}{d\delta}}_{\text{Internalized}} + \underbrace{\frac{\partial C_{2R}}{\partial \delta} - \frac{\partial T_2}{\partial R_1}\frac{dR_1}{d\delta} - \sum_j \frac{\partial T_2}{\partial \psi_j}\frac{d\psi_j}{d\delta}}_{\text{Not internalized}}$$

combined with optimality conditions, to show that

$$\frac{\frac{dW}{d\delta}}{1-\lambda} = \int_{\underline{s}}^{\hat{s}} U'(C_{2R})\left(\frac{\partial C_{2R}}{\partial \delta} - \frac{\partial T_2}{\partial R_1}\frac{dR_1}{d\delta} - \sum_j \frac{\partial T_2}{\partial \psi_j}\frac{d\psi_j}{d\delta}\right) dF(s)$$
$$+ \int_{\hat{s}}^{s^*} U'(C_{2R})\left(\frac{\partial C_{2R}}{\partial \delta} - \frac{\partial T_2}{\partial R_1}\frac{dR_1}{d\delta} - \sum_j \frac{\partial T_2}{\partial \psi_j}\frac{d\psi_j}{d\delta}\right) \pi dF(s)$$
$$+ \left[U(C_{2R}(s^*(\delta, R_1))) - U(C_{2N}(s^*(\delta, R_1)))\right] \pi \frac{\partial s^*(\delta, R_1)}{\partial \delta} f(s^*)$$

Using the same normalization with respect to $\frac{\partial q}{\partial \delta}$ as in the basic framework, we have that

$$\frac{\frac{dW}{d\delta}}{1-\lambda} = \left[U(C_{2R}(s^*)) - U(C_{2N}(s^*))\right]\frac{\partial q}{\partial \delta} + q\mathbb{E}_R\left[\frac{\partial C_{2R}}{\partial \delta} - \frac{\partial T_2}{\partial R_1}\frac{dR_1}{d\delta} - \sum_j \frac{\partial T_2}{\partial \psi_j}\frac{d\psi_j}{d\delta}\right]$$

The result follows directly using the same logic as above.

Proposition 5. (Optimal DI with alternative equilibrium selection)

Banks choose R_1 at date 0 maximizing

$$J(R_1; \delta) = \lambda U(R_1 D_0 + Y_1) + (1-\lambda)\left[\int_{\underline{s}}^{s^G(\delta, R_1)} U(C_{2R}(s, \delta, R_1))\, dF(s) + \int_{s^G(\delta, R_1)}^{\bar{s}} U(C_{2N}(s, R_1))\, dF(s)\right],$$

with R_1 choosing optimally according to

$$\left.\frac{\partial J}{\partial R_1}\right|_{T_2} = \lambda U'(R_1 D_0 + Y_1) D_0$$
$$+ (1-\lambda)\left[\int_{\underline{s}}^{s^G(\delta, R_1)} U'(C_{2R}(s, \delta, R_1))\frac{\partial \tilde{C}_{2R}}{\partial R_1} dF(s) + \int_{s^G(\delta, R_1)}^{\bar{s}} U'(C_{2N}(s, R_1))\frac{\partial C_{2N}}{\partial R_1} dF(s)\right]$$

Defining $W(\delta)$ as in our basic model, we can thus write

$$\frac{\frac{dW}{d\delta}}{1-\lambda} = \begin{array}{c} \int_{\underline{s}}^{s^G(\delta,R_1)} U'(C_{2R}) \left(\frac{\partial C_{2R}}{\partial \delta} - \frac{\partial T_2}{\partial R_1}\frac{dR_1}{d\delta}\right) dF(s) \\ + \left[U\left(C_{2R}\left(s^G(\delta,R_1)\right)\right) - U\left(C_{2N}\left(s^G(\delta,R_1)\right)\right)\right] \frac{\partial s^G(\delta,R_1)}{\partial \delta} f\left(s^G\right) \end{array}$$

Note that $q = F\left(s^G\right) = \int_{\underline{s}}^{s^G(\delta,R_1)} dF(s)$ and $\frac{\partial q}{\partial \delta} = \frac{\partial s^G(\delta,R_1)}{\partial \delta} f\left(s^G\right)$. As in the proof of proposition 2, equation (20) follows directly.

Proposition 5. (Optimal DI with aggregate spillovers)

We define two types of thresholds. The thresholds used by banks ex-ante to choose R_1 are denoted by $\hat{s}_0(R_1)$ and $s_0^*(\delta, R_1)$. Those perceived by the deposit insurance authority, incorporating the effects on aggregate withdrawals $\overline{\Delta}$, are denoted by $\hat{s}_1(R_1)$ and $s_1^*(\delta, R_1)$. However, in equilibrium, $\hat{s}_0(R_1) = \hat{s}_1(R_1)$ and $\hat{s}_0(\delta, R_1) = s_1^*(\delta, R_1)$. Hence, the actual probability of bank failure is independent of which threshold we use.

Social welfare can be written as in equation (11), with the exception that the limits of integration for the planner are now denoted by $\hat{s}_1(R_1)$ and $s_1^*(\delta, R_1)$Note that the planner uses different thresholds than. Because we assume that $\theta(\cdot) - 1$ represents a transfer, there is no need to modify C_{2R} and C_{2N}. Hence, we can write

$$\begin{aligned} \frac{dW}{d\delta} &= \lambda U'\left(R_1(\delta) D_0 + Y_1\right) D_0 \frac{dR_1}{d\delta} + (1-\lambda) \int_{\underline{s}}^{\hat{s}_1} U'(C_{2R}) \frac{dC_{2R}}{d\delta} dF(s) \\ &+ (1-\lambda) \int_{\hat{s}_1}^{s^*} \left(\pi U'(C_{2R}) \frac{dC_{2R}}{d\delta} + (1-\pi) U'(C_{2N}) \frac{dC_{2N}}{dR_1}\frac{dR_1}{d\delta}\right) dF(s) \\ &+ (1-\lambda) \int_{s^*}^{\overline{s}} U'(C_{2N}) \frac{dC_{2N}}{dR_1}\frac{dR_1}{d\delta} dF(s) \\ &+ (1-\lambda) \left[\begin{array}{c} (1-\pi)\left[U\left(C_{2R}\left(\hat{s}_1(R_1)\right)\right) - U\left(C_{2N}\left(\hat{s}_1(R_1)\right)\right)\right] \frac{\partial \hat{s}_1(R_1)}{\partial R_1}\frac{dR_1}{d\delta} f\left(\hat{s}_1\right) \\ +\pi\left[U\left(C_{2R}\left(s_1^*(\delta,R_1)\right)\right) - U\left(C_{2N}\left(s_1^*(\delta,R_1)\right)\right)\right] \left[\frac{\partial s_1^*(\delta,R_1)}{\partial R_1}\frac{dR_1}{d\delta} + \frac{\partial s_1^*(\delta,R_1)}{\partial \delta}\right] f\left(s_1^*\right) \end{array}\right] \end{aligned}$$

From now on, we do not distinguish with a subscript unless it is necessary. Decomposing $\frac{dC_{2R}}{d\delta}$, combined with the optimality condition on the deposit rate offered by banks, we can write

$$\frac{\frac{dW}{d\delta}}{1-\lambda} = \begin{array}{c} \int_{\underline{s}}^{\hat{s}} U'(C_{2R}) \left(\frac{\partial C_{2R}}{\partial \delta} - \frac{\partial T_2}{\partial R_1}\frac{dR_1}{d\delta}\right) dF(s) + \pi \int_{\hat{s}}^{s^*} U'(C_{2R}) \left(\frac{\partial C_{2R}}{\partial \delta} - \frac{\partial T_2}{\partial R_1}\frac{dR_1}{d\delta}\right) dF(s) \\ + \left[U\left(C_{2R}\left(s^*(\delta,R_1)\right)\right) - U\left(C_{2N}\left(s^*(\delta,R_1)\right)\right)\right] \pi \frac{\partial s^*(\delta,R_1)}{\partial \delta} f\left(s^*\right) \\ + (1-\pi)\left[U\left(C_{2R}\left(\hat{s}_1(R_1)\right)\right) - U\left(C_{2N}\left(\hat{s}_1(R_1)\right)\right)\right] \left(\frac{\partial \hat{s}_1(R_1)}{\partial R_1} - \frac{\partial \hat{s}_0(R_1)}{\partial R_1}\right) \frac{dR_1}{d\delta} f\left(\hat{s}_1\right) \\ +\pi\left[U\left(C_{2R}\left(s_1^*(\delta,R_1)\right)\right) - U\left(C_{2N}\left(s_1^*(\delta,R_1)\right)\right)\right] \left(\frac{\partial s_1^*(\delta,R_1)}{\partial R_1} - \frac{\partial s_0^*(\delta,R_1)}{\partial R_1}\right) \frac{dR_1}{d\delta} f\left(s_1^*\right) \end{array}$$

Note that we have two additional terms which were not present in the basic framework. They correspond to the spillovers effects which are not internalized by banks ex-ante when choosing R_1, but that the regulator takes into account.

Proposition 4. (Optimal DI with ex-ante heterogeneous depositors)

(To be included)

Proofs: Section 5

Proposition 8. (An approximation for δ^*)

The expression for δ^* involves the difference in utility between the consumption of depositors between failure and no failure equilibria at the threshold s^*: $U\left(C_{2R}\left(s^*\right)\right) - U\left(C_{2N}\left(s^*\right)\right)$. In general, this term is hard to map to observables because it is not written in marginal terms. However, we can bound it above and below by approximating that term around either C_{2N} or C_{2R}. For instance, we can write

$$U\left(C_{2R}\left(s^*\right)\right) - U\left(C_{2N}\left(s^*\right)\right) \approx U'\left(C_{2R}\left(s^*\right)\right)\left(C_{2N}\left(s^*\right) - C_{2R}\left(s^*\right)\right)$$

or

$$U\left(C_{2R}\left(s^*\right)\right) - U\left(C_{2N}\left(s^*\right)\right) \approx U'\left(C_{2N}\left(s^*\right)\right)\left(C_{2R}\left(s^*\right) - C_{2N}\left(s^*\right)\right)$$

Starting from our result in proposition 3, and using the first approximation, we can write δ^* as

$$\delta^* \approx \frac{\varepsilon_\delta^q\left(C_{2N}\left(s^*\right) - C_{2R}\left(s^*\right)\right)}{q\left(1+\kappa\right)\mathbb{E}_R\left[\frac{U'(C_{2R}(s))}{U'(C_{2R}(s^*))}\left(1-\chi\rho_2\left(s\right)\right)\right]},$$

where we assume that, in practice, at the optimum $\delta < \frac{r_1 D_0}{1-\lambda}$. Note that, when $\mathbb{E}_R\left[\frac{U'(C_{2R}(s))}{U'(C_{2R}(s^*))}\left(1-\chi\rho_2\left(s\right)\right)\right] = 1$, we can find the implied elasticity using $\varepsilon_\delta^q = \frac{\delta^* q(1+\kappa)}{C_{2N}-C_{2R}}$.

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Numerical example

As emphasized throughout the paper, especially in section 5, we exploit directly the sufficient statistics identified in the paper to think about quantification. However, it is instructive to illustrate the mechanics of the model with a numerical example. We solve our basic framework using the parameters given in table 2. This is not a calibration exercise and merely an illustration of the analytical results derived in the paper. In this example, we solve the problem with perfect ex-ante instruments. The qualitative insights when ex-ante instruments are not available do not change.

$\gamma = 10$	$Y_2(s) = 65$	$Y_1 = 50$	$D_0 = 30$	$\underline{s} = 0.95$
$\kappa = 0.16$	$\lambda = 0.23$	$\pi = 0.95$	$\chi = 0.75$	$\bar{s} = 1.8$

Table 2: Parameters numerical example

In particular, we assume that depositors have CRRA utility with an elasticity of intertemporal substitution given by $\frac{1}{\gamma}$. We also assume that the aggregate state is uniformly distributed following $s \sim U[\underline{s}, \bar{s}]$, and that the date 2 return on banks investments is given by $\rho_2(s) = s$. Using equations (4) and (5) in the text, we can explicitly write thresholds $\hat{s}(R_1)$ and $s^*(R_1, \delta)$ as

$$\hat{s}(R_1) = \frac{1}{1 - \frac{r_1}{R_1}\frac{1}{1-\lambda}} = \frac{(1-\lambda)R_1}{1 - \lambda R_1} \geq 1$$

$$s^*(R_1, \delta) = \frac{1}{1 - \frac{1}{\min\{\delta, D_0 R_1\}}\frac{r_1 D_0}{1-\lambda}} = \frac{1-\lambda}{1 - \lambda - \underbrace{\frac{r_1}{1+r_1}}_{<1}\underbrace{\frac{D_0 R_1}{\min\{\delta, D_0 R_1\}}}_{<1}} = \frac{(1-\lambda)R_1}{1 - \lambda R_1 + r_1\left(1 - \frac{D_0 R_1}{\min\{\delta, D_0 R_1\}}\right)}$$

It is easy to see that $\hat{s}(R_1) \geq 1$ when $R_1 \geq 1$, with equality when $R_1 = 1$. In this economy, we find that the optimal deposit rate chosen by banks is $R_1^*(\delta^*) = 1.12$ and the optimal level of deposit insurance is $\delta^* = 21.3$.

The left plot in figure 8 shows the return on bank investments and the right plot shows the level of consumption when there is a bank failure and where there is no bank failure for a given realization of s. We use the values of $\delta = 21.3$ and $R_1 = 1.12$, which will be the optimal values chosen in equilibrium.

The left plot in figure 9 is the analogous to figure 3 in the text and shows the different regions of equilibria for different values of δ. We use the value of $R_1 = 1.12$. The right plot in figure 9 shows the thresholds s^* and $\hat{s}$ as a function of the level of the deposit rate R_1, for $\delta = 21.3$.

Figure 10 shows the value of $J(R_1)$ used by the policymaker to determine the optimal level of R_1. We use again the value of $\delta = 21.3$ for illustration. The optimal R_1^* is the arg max of $J(R_1)$.

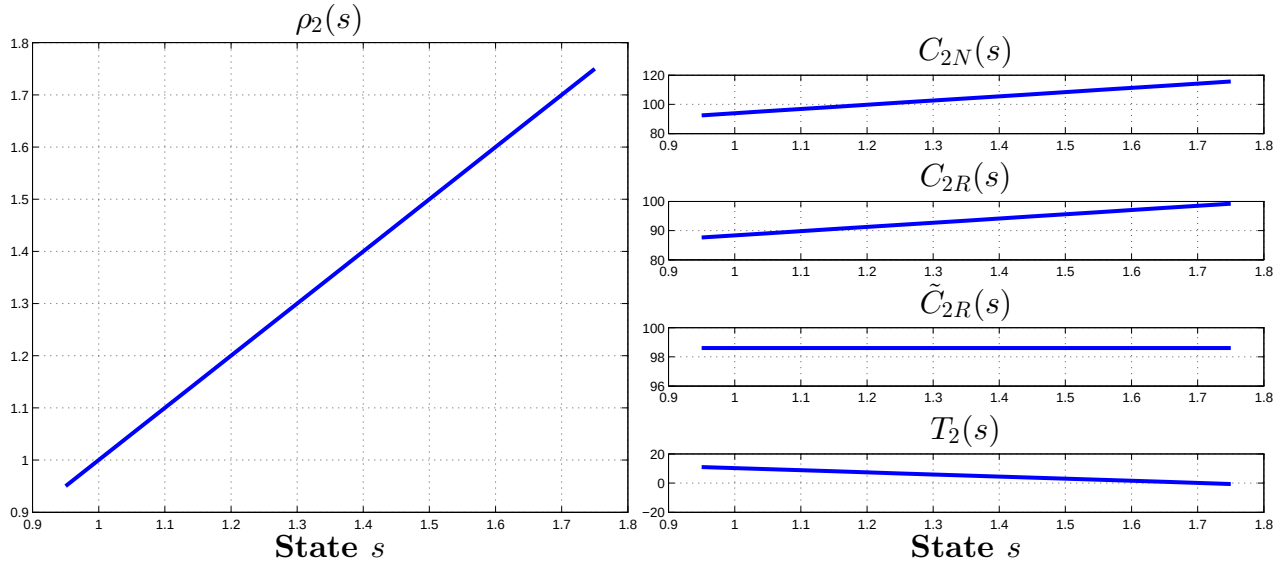

Figure 8: Returns on bank investments and consumption determinants given s

The upper left plot in figure 11 shows the value of early types utility in equation (9), while the lower left plot shows the value of the expected utility of late types. The three right plots show the value of the three components of late depositors' utility, as defined inside the bracket in equation (11).

The left plot in figure 12 shows social welfare for different levels of δ. The right plot in figure 12 shows the optimal level of R_1 for different values of δ.

We numerically corroborate our analytical result that showed that $\lim_{\delta \to 0^+} \frac{dW}{d\delta} < 0$. Intuitively, when δ is too low, it has no impact preventing bank failures. However, when the realization of s is sufficiently bad, the deposit insurance authority still has to tax depositors to pay for deposit insurance. Hence, as long as $\kappa > 0$, welfare is decreasing for low values of δ. In the region between the kinks in $W(\delta)$, the choice of R_1 is interior, which makes valid the first order approach used in the paper. Note that when δ is very low, banks offer very high deposit rates, because their choice of R_1 does not influence the likelihood of bank failure.

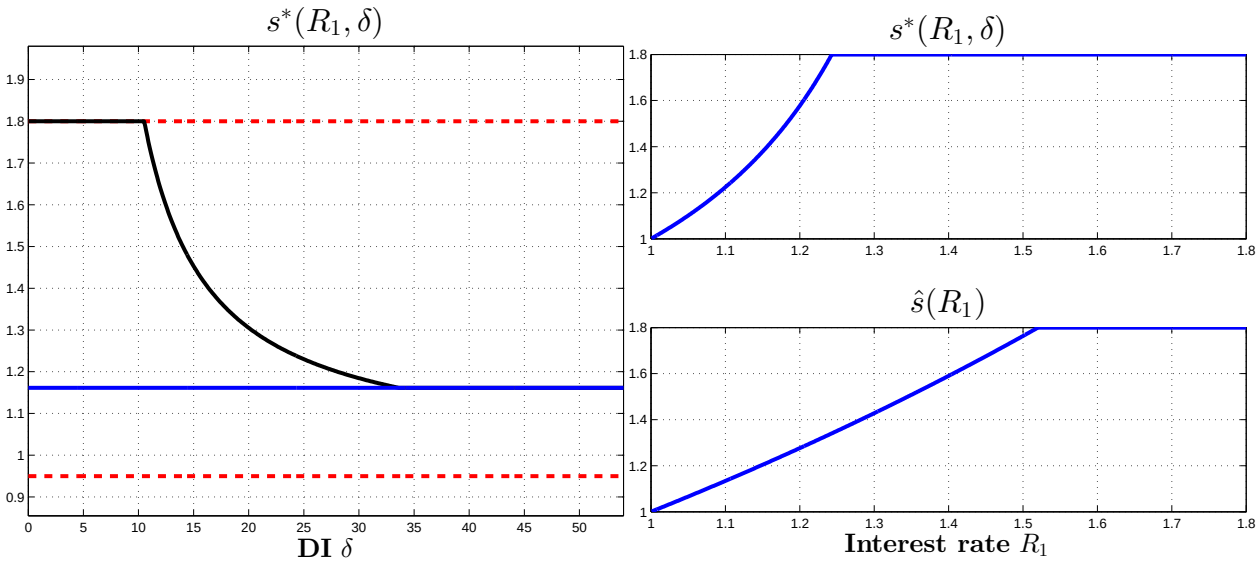

Figure 9: Thresholds $s^*(\delta, R_1)$ and $\hat{s}(R_1)$

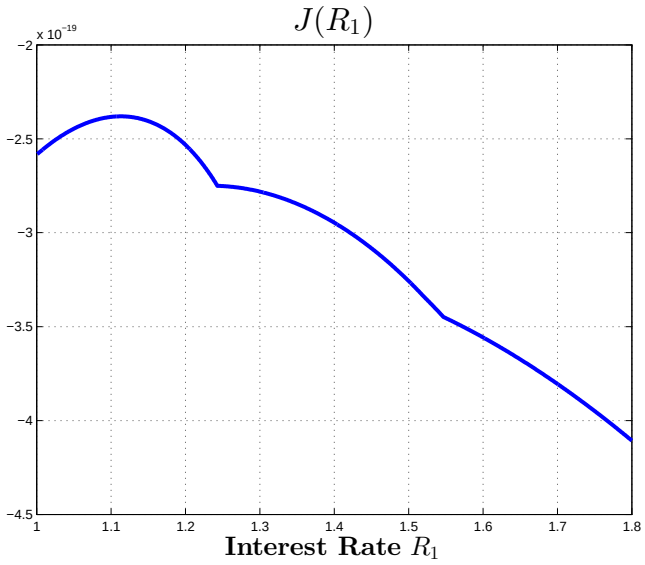

Figure 10: Value of $J(R_1)$

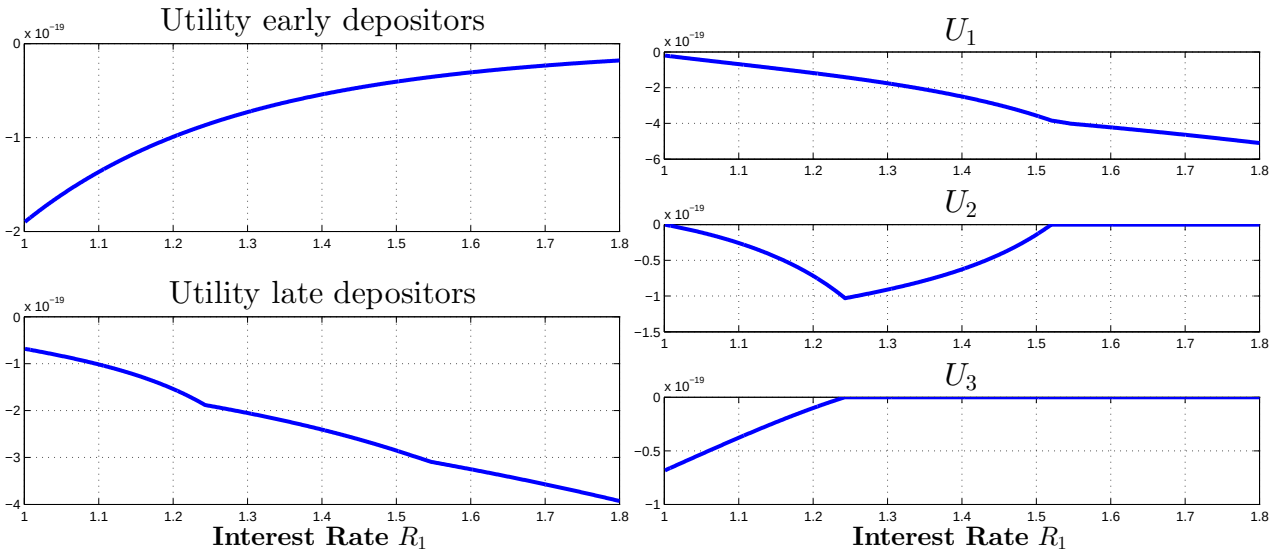

Figure 11: Decomposing $J(R_1)$

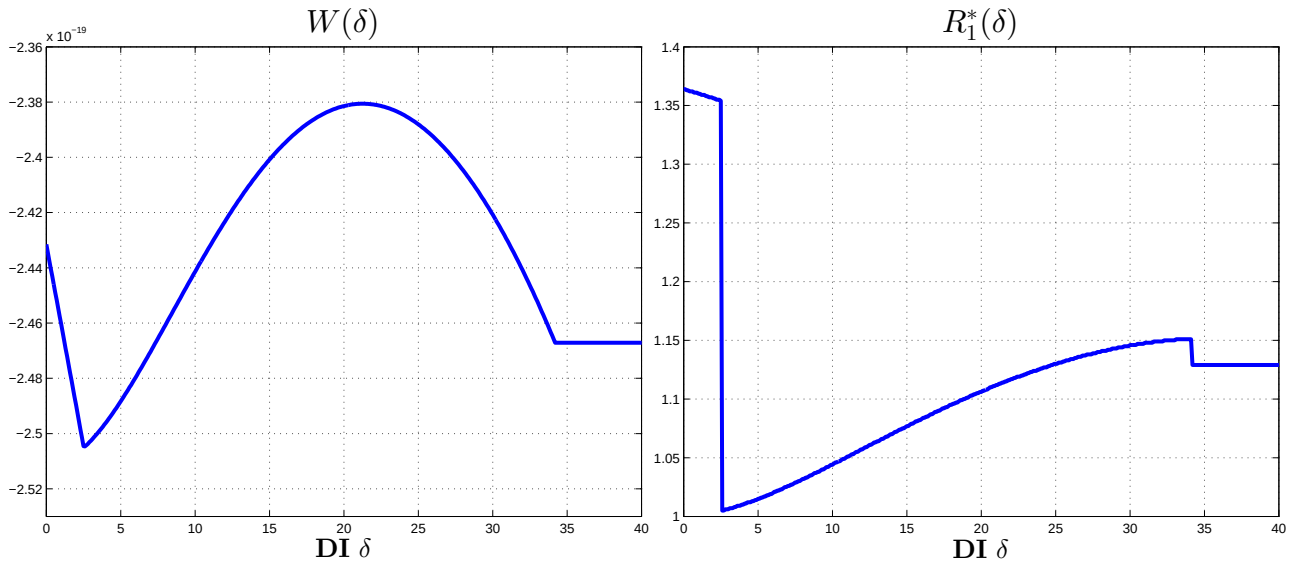

Figure 12: Social welfare $W(\delta)$ and optimal $R_1^*(\delta)$